Selective advantage of tolerant cultural traits in the Axelrod-Schelling model

C. Gracia-Lázaro,[1] L. M. Floría,[2,1,*] and Y. Moreno[2,3,†]
[1]Departamento de Física de la Materia Condensada, University of Zaragoza, Zaragoza, E-50009, Spain
[2]Institute for Biocomputation and Physics of Complex Systems (BIFI), University of Zaragoza, Zaragoza, E-50009, Spain
[3]Departamento de Física Teórica, University of Zaragoza, Zaragoza, E-50009, Spain
(Received 30 November 2010; published 4 May 2011)

The Axelrod-Schelling model incorporates into the original Axelrod's model of cultural dissemination the possibility that cultural agents placed in culturally dissimilar environments move to other places, the strength of this mobility being controlled by an intolerance parameter. By allowing heterogeneity in the intolerance of cultural agents, and considering it as a cultural feature, i.e., susceptible of cultural transmission (thus breaking the original symmetry of Axelrod-Schelling dynamics), we address here the question of whether tolerant or intolerant traits are more likely to become dominant in the long-term cultural dynamics. Our results show that tolerant traits possess a clear selective advantage in the framework of the Axelrod-Schelling model. We show that the reason for this selective advantage is the development, as time evolves, of a positive correlation between the number of neighbors that an agent has in its environment and its tolerant character.

DOI: 10.1103/PhysRevE.83.056103 PACS number(s): 89.75.Fb, 87.23.Ge, 89.20.−a

I. INTRODUCTION

Agent-based modeling is one of the most successful methodologies in recent social and economics research [1]. Its aim is "to discover fundamental local or micro mechanisms that are sufficient to generate the macroscopic social structures and collective behaviors of interest" [2]. Even very simple assumptions in the modeling often have consequences in the collective behavior that are far from obvious. The term agent indicates an interacting entity (e.g., individual, group, or institution) characterized by a set of internal states. The agent actual internal state may vary on time as an effect of the interactions with other agents. The dynamical social phenomena of interest include residential segregation [3,4], cultural globalization [5], opinion formation [6], rumor spreading [7,8], and others (see, e.g., Ref. [9]).

On the one hand, agent-based models (ABM) of social dynamics can use the statistical physics concepts and methods as an appropriate toolbox to speed up both comprehension and model predictions. On the other hand, these models are of interest in the field of nonequilibrium phase transitions in lattice models [10,11], as other stochastic spatial and network models motivated by population dynamics, epidemiology, or evolutionary biology [12].

The Axelrod's model for cultural dissemination attempts to understand the mechanisms underlying the tension between multiculturalism and cultural globalization, i.e., it addresses the question of why cultural differences between individuals and groups could persist despite the tendencies to become increasingly more similar as a result of social interactions. The model assumes a highly nonbiased scenario and culture is defined by a certain number of (equally important) cultural features capable of transmission among agents. The driving force of this cultural dynamics is the "homophile satisfaction" that plays the role of a utility function: In their social interactions the agents attempt to become more similar to their neighbors. Moreover, the likelihood that a cultural feature will spread from an agent to another depends on how many other features they already share, so the more similar two individuals are, the more similar they tend to become.

More specifically, in the Axelrod's model the culture of an agent is a vector of F integer variables $\{\sigma_f\}$ ($f = 1,\ldots,F$), called cultural *features*, that can assume q values, $\sigma_f = 0,1,\ldots,q-1$. At each elementary dynamical step, a randomly chosen agent i imitates an uncommon feature's trait of a randomly chosen neighbor j with a probability equal to their cultural overlap ω_{ij}, defined as the proportion of common cultural features,

$$\omega_{ij} = \frac{1}{F}\sum_{f=1}^{F}\delta_{\sigma_f(i),\sigma_f(j)}, \tag{1}$$

where $\delta_{x,y}$ stands for the Kronecker's δ which is 1 if $x = y$ and 0 otherwise. The mean cultural overlap $\bar{\omega}_i$ of an agent i with its k_i neighbors, defined as

$$\bar{\omega}_i = \frac{1}{k_i}\sum_{j=1}^{k_i}\omega_{ij}, \tag{2}$$

does not necessarily increase after a successful interaction (imitation) with one of the neighboring agents, for it will decrease if the changed feature was previously shared with two (or more) other neighbors. This renders the Axelrod's cultural dynamics nontrivial. This dynamics converges to a global monocultural macroscopic state when the initial cultural diversity q is below some critical value, whereas above it the homophilic social influence is unable to enforce cultural homogeneity, and the system freezes in a multicultural pattern. This change of macroscopic behavior has been characterized [9,13–17] as a nonequilibrium phase transition. The usual order parameter for the model is $\langle S_{\max}\rangle/N$, where $\langle S_{\max}\rangle$ is the average (over a large number of different random initial conditions) of the number of agents sharing the most abundant (dominant) culture and N is the number of agents in the population.

*mario.floria@gmail.com
†yamir.moreno@gmail.com

Several studies from the statistical physics perspective have analyzed the effects on the globalization-multiculturalism transition of diverse lattice or network structures [16–18], the presence of different types of noise ("cultural drift") [19,20], as well as the consideration of external fields (influential media, information feedback) [21–24] and global or local nonuniform couplings [25,26]. Coevolutionary dynamics of cultural traits and network of interactions has also been considered in Refs. [27,28].

An investigation on the effects of agents' mobility on cultural transmission was reported in Ref. [29] by considering that the agents move following the gradient of a "sugar" landscape (that they consume) and interact with agents nearby, so the driving force of agents' mobility is a new utility function (sugar consumption) that enters into the scene, whose details (as well as those of the landscape) need to be specified. Recently, a different setting regarding mobility has been considered [30], in which the driving force of mobility is the agents' cultural dissimilarity with their environment, i.e., homophile (dis-)satisfaction, the same that drives cultural transmission. In this model, two new parameters are introduced, namely the density h of empty lattice sites (places that are available to moving agents), and an intolerance parameter θ that controls the strength of the mobility: If an attempt to cultural interaction (imitation) fails, then the agent i moves to a randomly chosen empty lattice site if its mean cultural similarity $\bar{\omega}_i < \theta$. θ here is a threshold for tolerance, in such a way that high values of θ characterize intolerant societies (and please note that Ref. [30] uses the symbol T instead of θ for this intolerance parameter). Note, additionally, that in the presence of a density of empty sites, the sum in Eq. (2) runs over neighboring agents, and not on neighboring sites, so k_i can take on the values 0, 1, ...4 for a square lattice geometry.

This mechanism for agents mobility, based on homophile satisfaction, was directly inspired by one of the earliest examples of ABM in social science research, the well-known Schelling model of urban segregation [3,4], that plays a prominent role in segregation studies and public economics research [31–33]. Thus, the resulting model for cultural transmission among mobile agents is referred to as the Axelrod-Schelling model. Let us be careful with terms and establish the extent to which the Axelrod-Schelling model is related to the original Schelling model of urban segregation, recently also studied from statistical physics perspectives and methods [34–38].

When $F = 1$ the overlap ω_{ij} is a two-valued function ($\omega_{ij} = 0$ or 1) so there is simply no chance for cultural imitation (Axelrod dynamics) and the trait value of an agent is no longer a *cultural* trait but, let us say, an *ethnic* one, meaning a constant of motion (as, e.g., the color of the agent's skin). The mobility rule "move whenever $\bar{\omega}_i < \theta$" then translates into "move whenever the proportion of neighbors of your type [same $\sigma_1(i)$ value] is less than the threshold θ (intolerance parameter)." This defines (a standard variant of) the Schelling model dynamics, with a myopic long-range move, and with compulsory moves of isolated agents, a rather sensible feature (though not always present in the abundant literature on the Schelling model) whose main effect on the segregation dynamics is to favor hole segregation and, correspondingly, agent aggregation. Our main focus here is, however, not as much on the aggregation-segregation tension underlying the Axelrod-Schelling model as in the cultural interaction (traits imitation) that is intertwined with that essential tension and its feedback effects on it.

The mobility of cultural agents in the Axelrod-Schelling model leads to an enhancement of the convergence to cultural globalization so the behavior of the order parameter scales with the number N of cultural agents and the transition to multiculturalism occurs only for finite populations. This is basically the only effect for very low values of the density h of empty sites. However, for values of the density $(1 - h)$ of the agents below the lattice percolation threshold, new collective behaviors appear: (a) A fragmented multicultural phase occurs at very low values of the initial cultural diversity q, characterized by the reach of different local consensuses in disconnected monocultural clusters. (b) When q increases, the system undergoes a first-order transition to cultural globalization, and (c) by futher increasing q, the transition to the genuine Axelrod's multicultural phase finally occurs.

In this paper, we extend the Axelrod-Schelling model by considering intolerance θ as a cultural feature, and then it is no longer a parameter (a property of the whole population) but an individual property of agents subjected to cultural transmission. Due to its influence on the dynamics through the rule of mobility, the question of whether certain traits of this feature are more likely to be present in the dominant culture makes sense, contrary to what occurs with the rest of cultural features, whose particular traits do not influence the dynamics and are thus selectively neutral.

We have performed extensive numerical simulations that implement different rules for the mobility of agents, whose results show unambiguously that tolerant traits possess a selective advantage over intolerant ones, i.e., they are better adapted for survival in the long-term dynamics. Furthermore, by a stochastic analysis we present arguments showing that the reason of this cultural evolutionary success of tolerant traits is the establishment in the population of a negative correlation between the number k_i of neighboring agents and the value θ_i of the agent intolerance. This is presented in Sec. III. In Sec. II, we reconsider the transition between fragmented multiculturalism and globalization, first analyzed in Ref. [30], by using an alternative scheme for mobility with homogeneous intolerance. This new scheme corresponds to the homogeneous version of one of the rules of mobility used in Sec. III (mobility by social rejection), so this helps in the interpretation of some of these results and, at the same time, it throws a new light on the understanding of the mechanisms triggering this transition. Finally, we summarize our results in Sec. IV.

II. THE TRANSITION FROM FRAGMENTED MULTICULTURALISM TO GLOBALIZATION REVISITED

One of the new phenomena that appear associated to the mixed Axelrod-Schelling social dynamics is the existence, for values of the density $(1 - h)$ of agents below the lattice percolation threshold, of a multicultural macroscopic phase at very low values of the initial cultural diversity q. In this regime, the processes of local cultural convergence are faster that the typical time scales at which mobility is able to

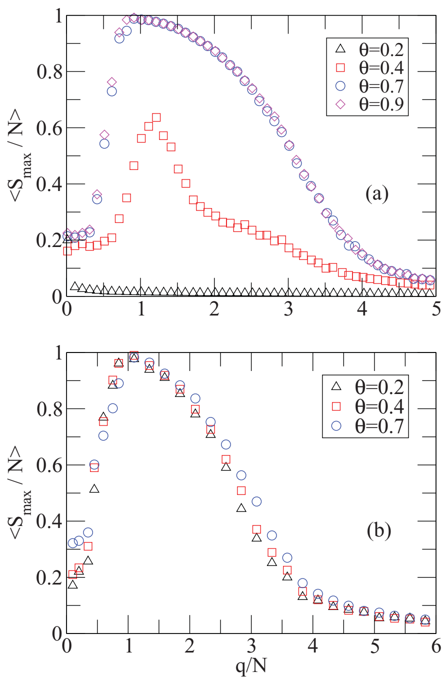

FIG. 1. (Color online) Order parameter $\langle S_{\max}\rangle/N$ versus scaled initial cultural diversity q/N for a density of empty sites $h = 0.5$ and lattice linear size $L = 40$. Panel (a) corresponds to scheme A for different values of the intolerance parameter. Panel (b) corresponds to scheme B. See the text for further details.

induce global convergence to a monocultural state. In this multicultural state agents are aggregated into disconnected (monocultural) clusters where different cultural consensuses have been achieved, hence the term *fragmented* for this multicultural phase.

If the value of q is increased [see Fig. 1(a)], the behavior for the order parameter $\langle S_{\max}\rangle/N$ becomes rather sensitive to the value of the intolerance parameter θ: For very low values of θ multiculturalism persists, while for very high values, a first-order transition to complete globalization is observed. At intermediate values of θ, the order parameter increases versus q but complete globalization is not reached. The observation that the increase of the initial cultural diversity promotes cultural globalization may seem paradoxical at a first sight, but it is not difficult to rationalize it by noting that an increase in q has also the effect of enhancing mobility, which is in turn an important driving force toward globalization. Moreover, insofar as higher values of θ enhance agents' mobility, the different behaviors that are observed for different values of the intolerance are consistent with this interpretation.

To deepen further our current understanding of the complex competing effects of different parameter variations that lead to the transition *fragmented multiculturalism-globalization*, we study here this transition in a different scheme for the mobility of cultural agents. We note here that in the original scheme of Ref. [30], after an elementary step of the Axelrod dynamics, if imitation has not occurred and $\omega_{ij} \neq 1$, the agent i moves to a randomly chosen empty site whenever $\bar{\omega}_i < \theta$. If the agent i turns out to be isolated, then it moves with certainty. We refer hereafter to this scheme as A. The mobility m_i of an agent i is defined as the probability that it moves in one elementary dynamical step (provided it has been chosen). Thus in scheme A:

$$m_i^A = (1 - \bar{\omega}_i)\, \Theta(\theta - \bar{\omega}_i), \tag{3}$$

where $\Theta(x)$ is the Heaviside step function that takes the value 1 if $x > 0$ and 0 if $x \leqslant 0$.

In the new scheme, hereafter referred to as B, after an elementary step of the Axelrod dynamics, if imitation has not occurred and $\omega_{ij} \neq 1$, the agent i moves to a randomly chosen empty site with probability $(1 - \bar{\omega}_i)\, \theta$. In the case where agent i is isolated, then it moves with certainty, as in the previous scheme. The mobility of agent i in scheme B is thus given by

$$m_i^B = (1 - \bar{\omega}_i)^2\, \theta. \tag{4}$$

Note that in both schemes the mobility is a decreasing function of $\bar{\omega}$. However, in scheme A the mobility vanishes in the interval $\bar{\omega} > \theta$ (being independent on θ for $\bar{\omega} < \theta$), whereas it does not vanish in scheme B, provided $\bar{\omega} \neq 1$ (and $\theta > 0$), though it takes lower values than in scheme A for $\bar{\omega} < \theta$ where it depends linearly on θ.

In Fig. 1(b) we plot the order parameter versus the scaled initial cultural diversity q/N for $h = 0.5$ and different values of the intolerance θ for scheme B and a two-dimensional square lattice geometry. In contrast with the results for scheme A [shown in Fig. 1(a)], the behavior of the order parameter turns out to be rather insensitive to the values of the intolerance θ, and the transition from the fragmented multicultural phase to globalization takes place for all the values of θ that we have used. Therefore, we have the following question regarding how to fit these observations into the interpretation framework given in Ref. [30] (succintly reproduced above in a previous paragraph) for the transition.

To have a better picture of the speed at which the processes of cultural convergence take place and what parameters are more influential on them, we have inspected the time evolution of the histograms of $\bar{\omega}$, namely $P(\bar{\omega},t)$, at values of the initial cultural diversity close (below and above) to the transition. In all cases and for both schemes, this probability density always evolves from being sharply concentrated near $\bar{\omega} = 0$ at $t = 0$ to become later widespread, the centroid shifting to progressively higher values of $\bar{\omega}$ as time goes by, until it concentrates near $\bar{\omega} = 1$, finally becoming a Dirac δ function $\delta(\bar{\omega} - 1)$. The time scale at which this evolution occurs seems not to be influenced by the scheme (A or B) adopted and the influence of the value of θ is also minor. The important parameter that mainly determines the time scale of local cultural convergence is the initial cultural diversity q: The lower its value, the faster this process takes place. What then makes a truly meaningful difference between, on the one hand, both scheme A at high θ values and scheme B at all θ values and, on the other hand, scheme A at low θ values (where the transition to globalization is absent) is that agents with high cultural overlap do not move in the latter.

These results throw new light onot the mechanisms that trigger the transition from the fragmented multicultural phase to cultural globalization. The increase of the initial cultural diversity slows down the local cultural convergence, thereby giving mobility a chance to induce global cultural consensus. But it is the mobility of agents with a significant high local cultural overlap (however small its mobility, as is the case for scheme B at low θ values), and not just the amount of overall mobility, that allows the effective cultural transmission among the disconnected clusters of the fragmented states, thus enabling the coalescence of the giant monocultural cluster characteristic of the globalization state. If mobility is strictly limited to culturally marginal agents, its power of cultural transmission is unable to overcome the fragmentation into disconnected cultural clusters.

III. HETEROGENEOUS INTOLERANCE.

As we have already mentioned in the Introduction, the mobility of cultural agents in the Axelrod-Schelling model is driven by the same utility (or social driving force) that underlies the cultural dynamics of the Axelrod model (as well as the dynamics of the Schelling model), namely "homophile satisfaction." In the model, those agents that are placed inside fully homogeneous cultural environments do not move. Cultural dissimilarities are the only source of mobility, and the parameter θ, which controls the strength of mobility, quantifies the degree of (in-)tolerance to cultural dissimilarities. Being a model parameter, tolerance is a quantity characteristic of the whole (artificial) society. In other words, in this context one can speak of tolerant (low value of θ) or intolerant societies. However, it seems to us rather natural to consider (artificial) societies where different agents have different degrees of tolerance to cultural dissimilarities. This certainly opens the possibility of new interesting questions to be investigated inside the model.

In what follows, we consider that each cultural agent i has assigned a real number $0 \leqslant \theta_i \leqslant 1$, called intolerance. Moreover, we are going to consider the intolerance of agents as a quantity associated to a *cultural feature*, i.e., a component of the cultural vector, and then subjected to temporal changes as a result of cultural interactions. Without loss of generality, one can associate the agents' intolerance to the first component σ_1 of the cultural vector $\{\sigma_f\}$. As this variable takes on integer $(0,1,\ldots,q-1)$ values, one has to choose some function $f(x)$ that takes values in the interval $[0,1]$ and define the intolerance θ_i of agent i to be

$$\theta_i = f(\sigma_1(i)). \tag{5}$$

We then have to specify the particular way in which the agents' intolerances enter into the dynamical rules. Many alternatives can indeed be considered for it, and our first choice will be the following: After an elementary step of the Axelrod dynamics, if imitation has not occurred and $\omega_{ij} \neq 1$, the agent i moves to a randomly chosen empty site with probability

$$\frac{1}{k_i}\sum_{j=1}^{k_i}(1-\omega_{ij})\theta_j, \tag{6}$$

where the sum extends to the k_i neighbors of i, and if the agent i is isolated ($k_i = 0$), it moves with certainty. In this choice, the intolerance θ_j of a cultural agent j is seen as its degree of hostility toward a culturally dissimilar neighbor i and is weighted by the cultural dissimilarity $(1-\omega_{ij})$. The mobility of an agent i is here the result of the *social rejection* of its neighbors, due to cultural dissimilarities.

The Axelrod-Schelling model with homogeneous tolerance, as the original Axelrod's model does, assumes an unbiased scenario in the sense that the traits of a cultural feature are completely interchangeable: Nothing in the dynamical rules distinguishes among different traits, and then the likelihood that each particular trait is present in the dominant culture of a realization is the same for all of them, provided they are uniformly distributed in the initial conditions for the dynamics. The particular traits that survive in the dominant culture of a given realization reach fixation by neutral selection, so, averaging over many independent realizations, one obtains a uniform distribution of traits in a large-enough sample of dominant cultures.

However, this symmetry of the model is broken in our current case of heterogeneous intolerance regarding the cultural feature σ_1, for its particular values do influence the local dynamics through the dynamical rule of mobility. The question of how likely different traits are to prevail and be present in the dominant culture now makes sense in this new symmetry-breaking scenario. Do tolerant traits possess a cultural selective advantage or, on the contrary, are intolerant traits better adapted to survive? Moreover, by which dynamical mechanisms are the "natural" selection of particular θ values built up in the time evolution of the populations of cultural agents?

Note that if one takes for $f(x)$ in Eq. (5) a constant function, so $\theta_i = \theta$ independent of i, one recovers scheme B, introduced in Sec. II. To the extent that the behavior of the order parameter $\langle S_{\max}\rangle/N$ (for a density of empty sites $h = 0.5$) in scheme B was seen to be rather insensitive to the value of θ, one should expect, in the present case of heterogeneous intolerance, that the order parameter for a density of empty sites $h = 0.5$ will be as shown in Fig. 1(b). Thus the choice made above in Eq. (6) is technically convenient for the purpose of investigating the question on the selective advantage of tolerant traits, just because it is expected that it leads to states of cultural globalization in some ranges of the initial cultural diversity, when the very term "dominant culture" is most meaningful.

We consider two-dimensional square lattices of linear size L, with periodic boundary conditions. The number F of cultural features is fixed to $F = 10$, and we have used two values of the density of empty sites, namely $h = 0.05$, to represent the situation in which agents percolate the lattice, and $h = 0.5$ to represent the opposite case. For $f(x)$ we will consider a simple linear function:

$$\theta_i = q^{-1}\sigma_1(i). \tag{7}$$

For the initial conditions, $N = (1-h)L^2$ agents are randomly distributed on the $L \times L$ lattice sites and randomly assigned a culture. The simulation of the cultural dynamics is stopped when the number of links for which $0 < \omega_{ij} < 1$, commonly called active links, vanishes. In addition to the order parameter, we compute the intolerance θ_D of the dominant culture, the average intolerance $\langle\theta\rangle$, and, sometimes, the histogram of intolerance values of the final state. The results that we show below are obtained by averaging over a large number (typically 10^3–10^4) of different initial conditions.

In the two panels of Fig. 2 we show our numerical results for $h = 0.05$ [Fig. 2(a)] and $h = 0.5$ [Fig. 2(b)]. First, we

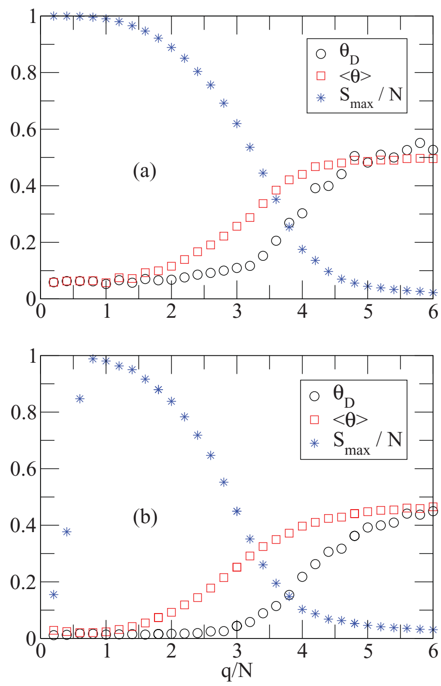

FIG. 2. (Color online) Order parameter $\langle S_{\max}\rangle/N$ (stars), intolerance θ_D of the dominant culture (circles), and average intolerance $\langle\theta\rangle$ (squares) versus scaled initial cultural diversity q/N for a lattice linear size $L = 40$, for the scheme of mobility corresponding to equation (6). Panel (a) corresponds to a density of empty sites $h = 0.05$. Panel (b) corresponds to $h = 0.5$. See the text for further details.

confirm the expectations on the behavior of the order parameter discussed above: Given the insensitive character of the order parameter in scheme B to the value of the intolerance parameter θ for both values of h, no effect on $\langle S_{\max}\rangle/N$ due to the heterogeneity of agents' intolerance is observed.

The numerical results for the intolerance values θ_D of the dominant culture for both values of the density of empty sites clearly show that very tolerant traits are better adapted to survive and become a part of the dominant culture. This occurs in the whole range of values of the initial cultural diversity that leads to values of the order parameter that are much larger than N^{-1} (so the term dominant possess a meaning). By comparing the graphs of θ_D shown in Figs. 2(a) and 2(b), we observe that the θ_D values are significantly lower for $h = 0.5$ than for $h = 0.05$, so the strength of the selective advantage of tolerant traits increases when the density h of empty sites is higher. The fact that the average intolerance $\langle\theta\rangle$ of the final configurations is higher than θ_D, provided the order parameter $N^{-1} \ll \langle S_{\max}\rangle/N < 1$, indicates that the nondominant surviving values of the intolerance are typically larger than the dominant one. We further show in Fig. 3 that the results regarding the behavior of θ_D and $\langle\theta\rangle$ for $L = 40$, are essentially unchanged for lattice of size 100×100.

In Fig. 4 we show the histogram of θ_D values, obtained from 2×10^3 realizations, at a fixed value of $q/N = 1.1$, for a density of empty sites $h = 0.05$. One should note that although the mean value of the dominant intolerance is at $\theta_D = 0.07$, the probability density is sharply peaked at $\theta_D = 0$ and quickly decays to negligible values as θ_D increases. In other words, the

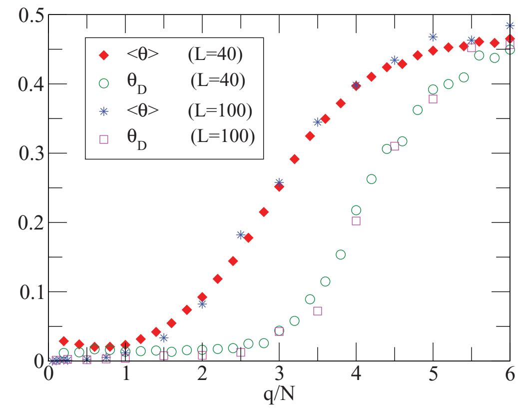

FIG. 3. (Color online) Comparison of T_D and $\langle T\rangle$ for $L = 40$ and $L = 100$, $h = 0.5$, using the same mobility rule in Fig 2.

lower the value of θ_D, the more probable it is, so the mean value is indicative only of the dispersion scale of the density.

In order to explain why tolerant traits are better adapted to prevail in the long term of the dynamics, let us consider the subset $A(\theta,t)$ of those cultural agents i for which, at time t, $\theta_i \leqslant \theta$, θ is an arbitrarily chosen value of the intolerance (e.g., $\theta = 0.3$, more or less). Let us denote by $n(\theta,t)$ the cardinal of $A(\theta,t)$ and call $\mathcal{L}(\theta,t)$ the set of lattice links (i,j) such that the agent i belongs to $A(\theta,t)$ and the agent j is not in this set (so $\theta_j > \theta$). If time is measured in elementary step units, the difference

$$\Delta n(\theta,t) = n(\theta,t+1) - n(\theta,t) \tag{8}$$

can only take on the values $0, \pm 1$. To compute the probability P_+ that $\Delta n(\theta,t)$ takes on the value $+1$, one has to sum the product of the following factors over all links $(i,j) \in \mathcal{L}(\theta,t)$:

(a) the probability (N^{-1}) of choosing agent j for a cultural imitation trial,

(b) the probability (k_j^{-1}) that its neighbor i is chosen,

(c) the probability (ω_{ij}) that agent j imitates an uncommon feature's trait of i, and

(d) the probability $\left[\frac{1}{(1-\omega_{ij})F}\right]$ that the chosen uncommon feature is σ_1.

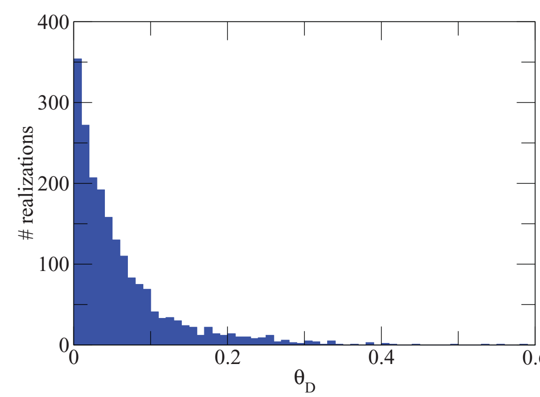

FIG. 4. (Color online) Histogram of the values of the intolerance θ_D of the dominant culture for 2×10^3 realizations, at scaled initial cultural diversity $q/N = 1.1$, and a density $h = 0.05$, of empty sites. See the text for further details.

Note that for a link (i,j) in the set $\mathcal{L}(\theta,t)$, the strict inequality $\omega_{ij} < 1$ holds. We then obtain

$$P_+ = \frac{1}{NF} \sum_{(i,j)\in\mathcal{L}(\theta,t)} \frac{1}{k_j} \frac{\omega_{ij}}{(1-\omega_{ij})}. \quad (9)$$

In a similar way, the probability P_- that $\Delta n(\theta,t)$ takes on the value -1 is

$$P_- = \frac{1}{NF} \sum_{(i,j)\in\mathcal{L}(\theta,t)} \frac{1}{k_i} \frac{\omega_{ij}}{(1-\omega_{ij})}. \quad (10)$$

We see that the number of agents in the set $A(\theta,t)$ performs a complicated random walk with left- and right-step probabilities changing in time as dictated by the model dynamics. The expected value of $\Delta n(\theta,t)$ is given by the difference $(P_+ - P_-)$, and then

$$E[\Delta n(\theta,t)] = \frac{1}{NF} \sum_{(i,j)\in\mathcal{L}(\theta,t)} \frac{(k_i - k_j)}{k_i k_j} \frac{\omega_{ij}}{(1-\omega_{ij})}. \quad (11)$$

This equation is the basis for an understanding of the selective advantage of tolerant traits. Indeed, following Eq. (6), agents with high θ_i values promote the mobility of their neighbors (leaving empty sites in their neighborhoods) more than do tolerant agents, so one should expect that a negative correlation between values of k_i and θ_i may be easily developed in the population, and tolerant agents may likely have larger values of k_i than those of intolerant agents. If this is the case, then Eq. (11) indicates that the random walk performed by $n(\theta,t)$ will be biased to the right, and the number of tolerant agents will likely increase as time evolves. The cultural selective advantage of tolerant traits has its origin on the bias produced by the negatively correlated degree of intolerance (k_i, θ_i) that is directly induced by the dynamical rule of social rejection.

Equation (11) also allows rationalization of the observation that the selective advantage of tolerant traits is strengthened by higher values of the density h of empty sites, because higher h values easily allow for higher values of the degree differences $(k_i - k_j)$ for $(i,j) \in \mathcal{L}(\theta,t)$ and so the bias favoring the increase of $n(\theta,t)$ can be stronger.

We have also considered a second way in which agents' intolerance enter into the mobility rule of the dynamics: After an elementary step of the Axelrod dynamics, if imitation has not occurred and $\omega_{ij} \neq 1$, the agent i moves to a randomly chosen empty site provided

$$\bar{\omega}_i < \theta_i. \quad (12)$$

Note that if one takes a constant function for $f(x)$ in equation (5), so $\theta_i = \theta$ independent of i, one recovers scheme A for homogeneous intolerance, which was used in Ref. [30]: The intolerance value is a threshold for the cultural overlap. But there is also an important difference here with respect to Eq. (6) regarding the interpretation, or meaning, of the intolerance. In Eq. (12), what determines whether an agent i moves is its own intolerance value θ_i and not its neighbors intolerance values, as in the previous case. Though both dynamical rules are based on homophile dissatisfaction, they in fact implement different plausible mechanisms for mobility. Whether the average social rejection (hostility) of my neighbors is more important than my own degree of tolerance with a dissimilar environment in the decision of moving may be a question with widely different (as well as context-dependent) individual answers, and it is certainly not within the scope of this paper to enter into such a discussion. We regard both here as alternative plausible mechanisms for mobility, which may lead to differences regarding the selective advantage of tolerant traits in the Axelrod-Schelling model with heterogeneous intolerance.

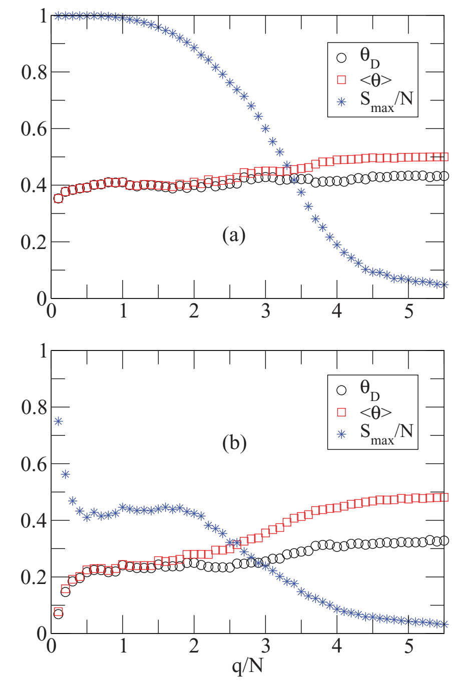

FIG. 5. (Color online) Order parameter $\langle S_{\max}\rangle/N$ (stars), intolerance θ_D of the dominant culture (circles), and average intolerance $\langle\theta\rangle$ (squares) versus scaled initial cultural diversity q/N for a lattice linear size $L = 40$ for the scheme of mobility corresponding to Eq. (12). Panel (a) corresponds to a density of empty sites $h = 0.05$. Panel (b) corresponds to $h = 0.5$. See the text for further details.

We show in Fig. 5 the results obtained for the dynamical rule associated to Eq. (12). Though the values of θ_D in this scheme are higher than those characteristic of the scheme analyzed before, a certain degree of selective advantage of tolerant traits is unambiguously observed. Also, the selective advantage is stronger for high density h of empty sites, as before. Now, however, agents move depending on their own intolerance values, and then it is not (at least) as clear as before whether a negative correlation between degree (k_i) and intolerance (θ_i) could be established, which would, in turn, explain the selective advantage of tolerant traits.

A possibility for this comes from the fact that intolerant agents move to empty sites more easily than tolerant agents do, so a negative (k_i, θ_i) correlation could appear, provided the lattice sites occupied by agents are more likely than empty sites to have agents in their neighborhood. To check for this, we have computed the time evolution of the average number of neighbors $\langle k\rangle$ of agents. Figure 6 shows that, after some (long) transient, the average degree of agents increases above its initial value [which is $\langle k\rangle = 4(1-h)$ for a square lattice and von Neumann neighborhood]. This increase of $\langle k\rangle$ corresponds to the coalescence of clusters that will become monocultural

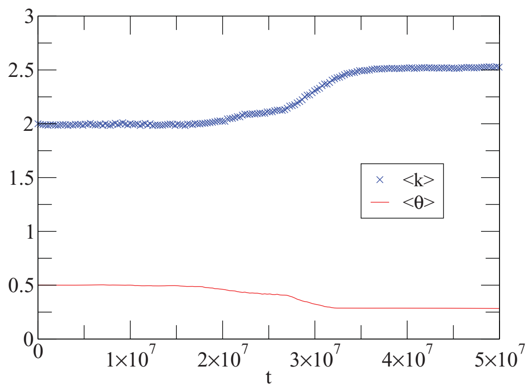

FIG. 6. (Color online) Time evolution of the average number of neighbors per agent $\langle k\rangle$ and average intolerance $\langle\theta\rangle$, for $q/N = 1$, $L = 40$, and $h = 0.5$ as obtained from 200 realizations in the scheme of Eq. (12).

in due (short) time. Interestingly, we also see in Fig. 5 the decrease of the average intolerance $\langle\theta\rangle$ as soon as the average degree increases, so giving further support to the argument.

Consequently, in the case where the agents' mobility is the result of their own intolerance to cultural dissimilarity, the tolerant traits possess selective advantage due to the establishment of a negative (k_i, θ_i) correlation that in this case has its origin in the agents' aggregation processes concomitant to the increase of local cultural overlaps. The observed fact that the selective advantage of tolerant traits is now weaker than in the case when mobility is induced by social rejection may likely be the effect of two confluent factors; on the one hand, the development of negative (k_i,θ_i) correlation is not a direct consequence of the dynamical rule and, on the other, as analyzed in previous Sec. II, agents' aggregation processes are much less effective when intolerance enters as a threshold for mobility.

IV. SUMMARY AND CONCLUDING REMARKS

In the Axelrod-Schelling model for cultural dissemination among mobile agents, we have considered intolerance, which was originally [30] a model parameter controlling the strength of agents' mobility, as a variable associated to a cultural feature and thus subjected to cultural transmission. We have performed extensive numerical simulations for two different dynamical rules for mobility, whose respective homogeneous versions are analyzed with respect to the transition from a topologically fragmented local consensus to a global cultural consensus that occurs at very low values of the initial cultural diversity. In the first of these dynamical rules (mobility by social rejection), agents move due to the intolerance of their neighbors, weighted by their cultural dissimilarity, whereas, in the second one, the mobility depends on the agent's own intolerance to the cultural dissimilarity with its environment. In both cases our results indicate that tolerant traits are selectively advantageous, so the intolerance values present in the dominant culture are preferentially low. One then sees how the breaking of the original symmetry (indifference of the dynamics with respect to a particular feature's trait values, which leads to purely neutral selection of dominant characters in cultural evolution) effectively allows for the appearance of natural selection of advantageous traits.

The selective advantage of tolerant traits increases with the density h of empty lattice sites and is also higher for the first scheme, where mobility is the result of the social rejection from the neighborhood. A stochastic analysis allows the rationalization of all these numerical observations and points to the dynamical development of a negative correlation between the number of neighbors of an agent and its intolerance value as the origin of the selective advantage of tolerant traits. We should emphasize here that regarding the rule of cultural imitation, nothing favors tolerant traits over intolerant ones, i.e., Axelrod's cultural interactions are completely unbiased, so the bias toward tolerant traits can come from only the influence of the tolerance cultural feature on the mobility of agents, which shapes the instantaneous network of interactions among cultural agents. One should expect findings for other network-updating dynamics that are analogous to the one considered (in the symmetric context) by Refs. [27,28] and that also show topologically fragmented phases, provided the trait symmetry is broken at the network-updating rule level.

In this regard, the term *tolerance*—in the context of the Axelrod-Schelling model—has a very precise and narrow meaning, one that is much more limited than its usual meaning in social science and political philosophy, where it is much more than just a conditioning factor of the mobility of individuals and groups. However, inside the limitations of a simple agent-based model like this one, our findings on the "adaptive to survival" character of tolerant traits in cultural dynamics point to basic mechanisms that can be highly influential in cultural evolution.

ACKNOWLEDGMENTS

This work has been partially supported by Spanish MICINN through projects FIS2008-01240 and FIS2009-13364-C02-01 and by Comunidad de Aragón (Spain) through a grant to the FENOL group.

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