EUROPHYSICS LETTERS 1 August 2003

Europhys. Lett., **63** (3), pp. 347–353 (2003)

Creep rupture has two universality classes

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(received 31 January 2003; accepted in final form 23 May 2003)

PACS. 46.50.+a – Fracture mechanics, fatigue and cracks.
PACS. 62.20.Mk – Fatigue, brittleness, fracture, and cracks.
PACS. 91.60.Dc – Creep and deformation.

Abstract. – Objects subject to a steady load will often resist it for a long time before weakening and suddenly failing. This process can be studied by fiber bundle models, in which fibers fail in a random fashion that depends both upon the integrity of their neighbors and the load to which they are subjected. In this letter, we introduce a new fiber bundle model that allows us to examine how the behavior of such a system depends upon the range over which each fiber interacts with its neighbors. Using analytical and numerical arguments, we show that for all systems there is a critical load below which the system reaches equilibrium, and above which it fails in finite time. We consider how the time to failure depends upon how much the applied load exceeds the critical load, and find two universality classes. For short-range interactions, failure is abrupt, and time to failure is independent of load. For long-range interactions, the time to failure depends upon the excess load as a power law. The transition between these two universality classes is sharp as a function of the range of interaction. In both universality classes, the distribution of times between breaking events obeys a power law as the system creeps towards failure.

Introduction. – Time evolution of physical systems under a steady external driving is abundant in nature. Examples can be found in seemingly diverse systems such as the behavior of domain walls in magnets [1], earthquake dynamics [2, 3] and creep rupture of solids [4–8]. Recent intensive experimental and theoretical studies revealed that, in spite of the diversity of the governing physical dynamics, the behaviors of these systems share several universal features: local stresses arising due to the external driving are resolved in the form of avalanches of microscopic relaxation events. As a result, the system attains a stationary macroscopic state characterized by a scale-invariant microscopic activity. In particular, large efforts have been devoted to the study of material failure occurring under various loading conditions [4–16] since these studies also provide direct information on the dynamics of earthquakes [2, 3, 8].

Under high steady stresses, materials may undergo time-dependent deformation resulting in failure called creep rupture which limits their lifetime, and hence, has a high impact on their applicability in construction elements. Creep failure tests are usually performed under uniaxial tensile loading when the specimen is subjected to a constant load σ_0 and the time evolution of the damage process is followed by recording the strain ε of the specimen and the acoustic signals emitted by microscopic failure events. During the last decade several theoretical approaches have been worked out to describe time-dependent deformation and rupture of materials occurring in a finite time [9–16]. In spite of the large amount of experimental and theoretical results accumulated, a comprehensive theoretical picture of creep rupture is still lacking.

In this letter we study the creep rupture of materials by means of a novel fiber bundle model. During the past years Fiber Bundle Models (FBMs) [13–24] of materials damage not only played a very important role in the study of fracture but they turned out to be also one of the most promising approaches to earthquake predictions [3, 25, 26]. Our model describes the interaction of fibers in terms of an adjustable load-sharing function covering all cases relevant to real materials; furthermore, it combines the viscoelastic constitutive behavior and breaking of fibers. Analytical and numerical calculations show that there exists a critical load that determines the final state of the material. Strikingly, we find that in creep rupture there are only two universality classes and that the distribution of times between breaking events shows a universal power law behaviour. The relevance of our results to experiments is discussed.

Viscoelastic fiber bundle. – Our model consists of N parallel fibers arranged on a square lattice of side length L having viscoelastic constitutive behavior [13]. For simplicity, the pure viscoelastic behavior of fibers is modeled by a Kelvin-Voigt element which consists of a spring and a dashpot in parallel and results in the constitutive equation $\sigma_0 = \beta\dot{\varepsilon} + E\varepsilon$, where σ_0 is the imposed load, β denotes the damping coefficient, and E the Young modulus of fibers, respectively. In order to capture failure in the model a strain-controlled breaking criterion is imposed, *i.e.* a fiber fails during the time evolution of the system when its strain exceeds a breaking threshold ε_i, $i = 1, \ldots, N$, drawn from a probability distribution $P(\varepsilon) = \int_0^{\varepsilon} p(x)\mathrm{d}x$. When a fiber fails, its load is redistributed to the intact fibers according to the interaction law of fibers. It was shown in ref. [13] that in a viscoelastic bundle the fibers break one-by-one, which implies that such a system is very sensitive to the details of load sharing, much more than a static fiber bundle [13–21], where a large number of fibers can break at once in the form of bursts [17]. In order to realistically model the stress transfer between fibers, recently an adjustable stress transfer function was introduced, which interpolates between the two limiting cases of global and local load sharing (GLS and LLS) [18, 19]. Motivated by mechanics-based models of fracture [23, 24] we assume that the additional load σ_{add} received by an intact fiber i on a square lattice after the failure of fiber j depends on their distance r_{ij} and has the form $\sigma_{\mathrm{add}} = Zr_{ij}^{-\gamma}$. Here the normalization factor Z is simply $Z = \sum_{i\in I} r_{ij}^{-\gamma}$ and the sum runs over the set I of intact fibers [21]. The exponent γ is an adjustable parameter of the model, which controls the effective range of load redistribution. The limiting cases $\gamma = 0$ and $\gamma \to \infty$ recover the global and local load sharing widely studied in the literature; furthermore, intermediate values of γ interpolate between them. For the breaking thresholds of fibers a uniform distribution between 0 and 1, and a Weibull distribution of the form $P(\varepsilon) = 1 - \exp[-(\varepsilon/\lambda)^{\rho}]$ were used with Weibull moduli $\rho = 2$ and 5. A comprehensive study of the quasistatic fracture of fiber bundles in terms of the adjustable load-sharing function has been presented in ref. [21].

Global load sharing. – In the mean-field limit, *i.e.* global load sharing obtained for $\gamma = 0$, many of the quantities describing the behavior of the system can be obtained analytically. In

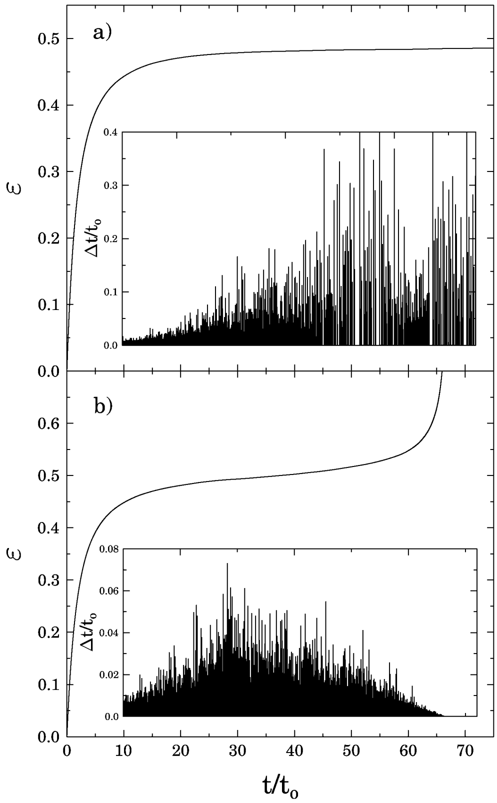

Fig. 1 – $\varepsilon(t)$ for two different values of the external load a) $\sigma_0 < \sigma_c$, b) $\sigma_0 > \sigma_c$. The insets present the inter-event times Δt at the time of their occurrence t.

this case, the time evolution of the system under a steady external load σ_0 is described by the differential equation

$$\frac{\sigma_0}{1 - P(\varepsilon)} = \beta\dot{\varepsilon} + E\varepsilon, \tag{1}$$

where the viscoelastic behavior of fibers is coupled to the failure of fibers [13]. For the behavior of the solutions $\varepsilon(t)$ of eq. (1), two distinct regimes can be distinguished depending on the value of the external load σ_0: When σ_0 falls below a critical value σ_c, eq. (1) has a stationary solution ε_s, which can be obtained by setting $\dot{\varepsilon} = 0$, *i.e.* $\sigma_0 = E\varepsilon_s[1 - P(\varepsilon_s)]$. It means that until this equation can be solved for ε_s at a given external load σ_0, the solution $\varepsilon(t)$ of eq. (1) converges to ε_s when $t \to \infty$, and the system suffers only a partial failure. However, when σ_0 exceeds the critical value σ_c, no stationary solution exists; furthermore, $\dot{\varepsilon}$ remains always positive, which implies that for $\sigma > \sigma_c$ the strain of the system $\varepsilon(t)$ monotonically increases until the system fails globally at a finite time t_f. The behavior of $\varepsilon(t)$ is illustrated in fig. 1 for two values of σ_0 below and above σ_c with uniformly distributed breaking thresholds. It follows from the above argument that the critical value of the load σ_c is the static fracture strength of the bundle [18]. In ref. [14] a similar overall behaviour was obtained for long fiber composites subjected to a constant load.

The creep rupture of the viscoelastic bundle can be interpreted so that for $\sigma_0 \leq \sigma_c$ the bundle is partially damaged implying an infinite lifetime $t_f = \infty$ and the emergence of a stationary macroscopic state, while above the critical load $\sigma_0 > \sigma_c$ global failure occurs at a finite time t_{f}, but in the vicinity of σ_{c} the global failure is preceded by a long-lived stationary state. The nature of the transition occurring at σ_{c} can be characterized by analyzing how the creeping system behaves when approaching the critical load both from below and above.

For $\sigma_0 \leq \sigma_{\mathrm{c}}$, the fiber bundle relaxes to the stationary deformation ε_{s} through a gradually decreasing breaking activity. It can be shown analytically that $\varepsilon(t)$ has an exponential relaxation to ε_{s} with a characteristic time scale τ that depends on the external load σ_0 as

$$\tau \sim (\sigma_{\mathrm{c}} - \sigma_0)^{-1/2}, \quad \text{for } \sigma_0 < \sigma_{\mathrm{c}}, \tag{2}$$

i.e., when approaching the critical point from below the characteristic time of the relaxation to the stationary state diverges according to a universal power law with an exponent $-1/2$ independent of the form of disorder distribution P. Note that a similar power law divergence of the number of successive relaxation steps of a dry fiber bundle subjected to a constant external load was pointed out in refs. [15, 16]. Above the critical point, the lifetime t_{f} defines the characteristic time scale of the system which can be cast in the form [13]

$$t_{\mathrm{f}} \sim (\sigma_0 - \sigma_{\mathrm{c}})^{-1/2}, \quad \text{for } \sigma_0 > \sigma_{\mathrm{c}}, \tag{3}$$

so that t_{f} also has a power law divergence at σ_{c} with a universal exponent $-1/2$ like τ below the critical point. Hence, for global load sharing $\gamma = 0$, the system exhibits scaling behavior on both sides of the critical point, indicating a continuous transition at the critical load σ_{c}.

In a creeping system, due to the steady external driving, local overloads build up slowly on the microscopic level when the breaking threshold of fibers is exceeded by the local deformation. The sudden breaking of fibers occurring on a time scale much shorter than the scale of the driving provides the relaxation mechanism which resolves the overloads in the system. This mechanism consists of sequential fiber breakings that form an avalanche which either stops (below the critical point) or continues until the whole system gets destroyed (above the critical point).

Fibers fail one by one, furthermore, under GLS conditions, breakings occur in the order of increasing breaking thresholds ε_i and the time $\Delta t(\varepsilon_i, \varepsilon_{i+1})$ elapsed between the breaking of i-th and $i+1$-th fibers can be analytically obtained. The inter-event time Δt is a fluctuating quantity which depends both on the breaking thresholds and the load level, as illustrated in the inset of fig. 1. It can be observed in the figure that before and after the plateau of $\varepsilon(t)$ the inter-event times are relatively short (low peaks), while along the plateau Δt is scattered over a broad interval. The statistics of inter-event times characterized by the distribution $f(\Delta t)$ provides information on the microscopic dynamics of creep. $f(\Delta t)$ is presented in fig. 2 for a system of $N = 2 \times 10^7$ fibers with uniformly distributed breaking thresholds. Simulations revealed that $f(\Delta t)$ exhibits a power law of the form $f(\Delta t) \sim \Delta t^{-b}$ both below and above the critical point whenever the macroscopic stationary state characterized by the plateau of $\varepsilon(t)$ is attained. However, the value of the exponent b is different on the two sides of the critical point, *i.e.* below σ_{c} the exponent of the distribution is $b = 1.95 \pm 0.05$ independent of σ_0, while above σ_{c} we obtained $b = 1.5 \pm 0.05$. Increasing the load above σ_{c} the stationary state gradually disappears, implying that the power law regime of $f(\Delta t)$ preceding the exponential cut-off is getting shorter but the exponent remains the same (see fig. 2). The emergence of a macroscopic stationary state accompanied by a power law distribution of inter-event times obtained in a certain range of load for creep rupture resembles scenarios that have been called self-organized criticality in the literature [27, 28].

Local load sharing. – To explore the effect of the details of load redistribution on the creep rupture process we studied how the behavior of the system changes in the vicinity of

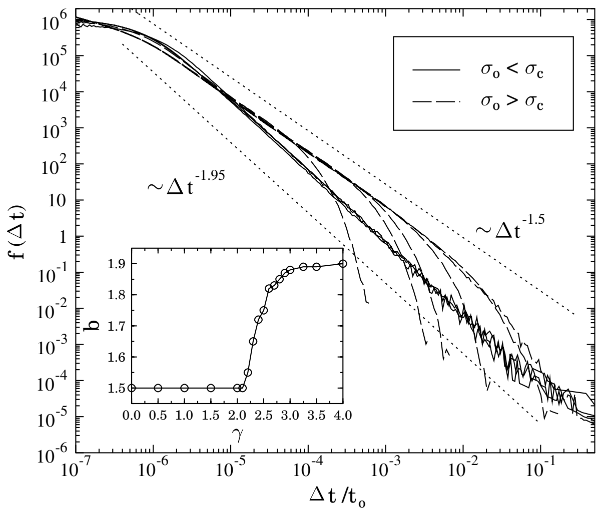

Fig. 2 – The distribution of inter-event times Δt for $\gamma = 0$. The power law behavior can be observed over 5 orders of magnitude. Inset: the exponent b of $f(\Delta t)$ as a function of γ for $\sigma_0 > \sigma_c$.

the critical point when the load sharing gets localized. Simulations have been performed by varying the effective range of interaction of fibers by controlling the exponent γ of the load-sharing function. The inset of fig. 3 presents the lifetime t_f of a bundle of fibers arranged on a square lattice of side length $L = 101$ with uniformly distributed breaking thresholds as a function of the distance from the critical point $\Delta\sigma = \sigma_0 - \sigma_c$ for several values of the exponent γ. It can be observed that the $t_f(\Delta\sigma; \gamma)$ curves form two groups of different functional form: The upper group is obtained for $0 \le \gamma \le 1.95$ when the load sharing is global due to the divergence of the normalization factor Z of the load-sharing function σ_{add} [21] and eq. (3) holds [13]. However, in the lower group, obtained for $\gamma > 2.9$ when the

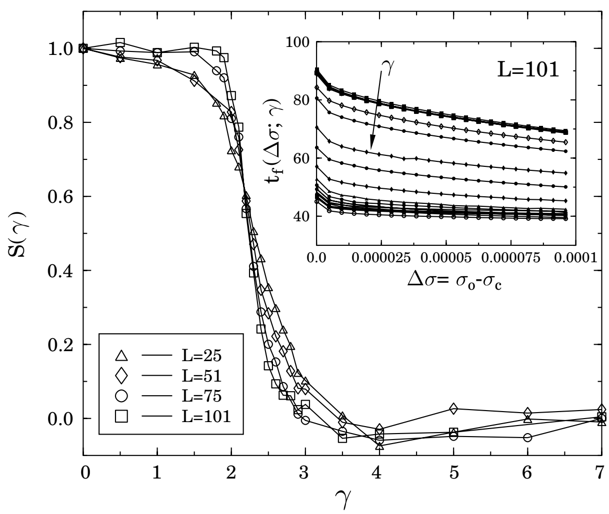

Fig. 3 – Inset: the lifetime t_f as a function of $\sigma_0 - \sigma_c$ for different values of the exponent γ between 0 and 100. $S(\gamma)$ is presented in the main figure for several system sizes L.

load sharing gets localized [21], $t_\mathrm{f}(\Delta\sigma;\gamma)$ rapidly takes a constant value showing an abrupt transition at the critical load σ_c with no scaling, reminiscent of a first-order transition. The results imply the existence of two universality classes in creep rupture characterized by a completely global (GLS), or a completely local (LLS) behavior depending on the effective range of interaction γ with a rather sharp transition between them. In order to quantify the behavior of $t_\mathrm{f}(\Delta\sigma;\gamma)$ under the variation of γ we calculated the normalized quantity $S(\gamma) = [t_\mathrm{f}(\gamma) - t_\mathrm{f}(\gamma = 10)]/[t_\mathrm{f}(\gamma = 0) - t_\mathrm{f}(\gamma = 10)]$, where $t_\mathrm{f}(\gamma)$ denotes the value of t_f at the smallest value of $\Delta\sigma$ used to calculate $t_\mathrm{f}(\Delta\sigma;\gamma)$ at a given γ. Figure 3 shows that $S(\gamma)$ provides a quantitative description of the creep rupture transition in terms of the effective range of interaction, so that $S(\gamma)$ takes value unity for the GLS, and it has a value close to zero for the LLS class, respectively. It can also be observed in fig. 3 that the transition between the two universality classes gets sharper around $\gamma_\mathrm{c} \approx 2$ with increasing system size. Real materials described by a finite value of γ must fall into one of the above universality classes. The existence of only two universality classes implies that the mean-field analytical results can be extended beyond $\gamma = 0$, *i.e.*, they apply for a wider interaction range which is relevant for real materials.

Extensive simulations revealed that whenever a macroscopic stationary state is attained by the system, the distribution of inter-event times follows a power law irrespective of the range of interaction γ. Below the critical point the exponent b of the distribution has a value $b = 1.95 \pm 0.05$ independent of γ, while above the critical point b is different in the two universality classes as illustrated by the inset of fig. 2. In the LLS class the exponent b has practically the same value below and above σ_c. Simulation performed with a uniform and Weibull distributions of parameter values $\rho = 2$ and 5 for the breaking thresholds showed that the value of the exponent b is insensitive to the details of the disorder distribution P.

The breaking process of fibers occurring in a solid under various loading conditions can be monitored by acoustic emission techniques which has also been applied to study creep rupture. The statistics of inter-event times has been studied in various types of materials like wood, plaster, basalt, and fiber glass. It was found experimentally that the distribution of inter-event times always exhibits a power law behavior; however, the values of b was found to depend on the material falling between 1.2 and 1.9 [4–8] for $\sigma_0 > \sigma_\mathrm{c}$. Hence, our theoretical findings are in a quite reasonable agreement with the available experimental results.

Conclusions. – We have identified two universality classes of creep rupture depending on the range of interaction of fibers: the creep rupture transition is either continuous, characterized by power law divergences, or abrupt with no scaling. We demonstrated analytically and numerically that in both universality classes the creeping system evolves into a macroscopic stationary state which is accompanied by the emergence of a power law distribution of waiting times between consecutive breaking events on the microscopic level. Our theoretical results provide a consistent explanation of recent experimental findings on the damage process of creep rupture.

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We thank Prof. M. Marder for kindly improving the presentation of our manuscript. YM thanks A. Vespignani for enlighting discussions and A. F. Pacheco for useful comments. This work was supported by the DFG project SFB 381 and by the NATO grant PST.CLG.977311. FK acknowledges the kind hospitality of the Condensed Matter Group of ICTP Trieste, and financial support of the Bólyai János and Békési György Fellowships of the Hungarian Academy of Sciences and of the Research Contracts FKFP 0118/2001 and T037212. YM acknowledges financial support from the Secretaría de Estado de Educación y Universidades (Spain, SB2000-0357) and of the Spanish DGICYT Project BFM2002-01798.

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