# Effects of mass media action on the Axelrod model with social influence

Arezky H. Rodríguez[1] and Y. Moreno[2,3]

[1]*Academia de Matemáticas, Universidad Autónoma de la Ciudad de México, Mexico City 03020, Mexico*
[2]*Instituto de Biocomputación y Física de Sistemas Complejos, Universidad de Zaragoza, Zaragoza 50009, Spain*
[3]*Departamento de Física Teórica, Facultad de Ciencias, Universidad de Zaragoza, Zaragoza 50009, Spain*

The use of *dyadic interaction* between agents, in combination with *homophily* (the principle that "likes attract") in the Axelrod model for the study of cultural dissemination, has two important problems: the prediction of monoculture in large societies and an extremely narrow window of noise levels in which diversity with local convergence is obtained. Recently, the inclusion of *social influence* has proven to overcome them [A. Flache and M. W. Macy, e-print arXiv:0808.2710]. Here, we extend the Axelrod model with social influence interaction for the study of mass media effects through the inclusion of a superagent which acts over the whole system and has non-null overlap with each agent of the society. The dependence with different parameters as the initial social diversity, size effects, mass media strength, and noise is outlined. Our results might be relevant in several socioeconomic contexts and for the study of the emergence of collective behavior in complex social systems.

## I. INTRODUCTION

Social human systems are constituted by human agents which interact with a limited number of peers [1]. This perhaps apparently random procedure yields global regularities: there are order-disorder transitions (spontaneous formation of cultures, the emergence of consensus about different issues, and spontaneous formation of a common language), examples of scaling and universality, to mention just a few. These phenomena are now been under intense studies through different techniques. One of them is the so-called agent-based modeling [2–7].

The interest in the application of such techniques has grown rapidly mainly as a result of the increasing availability of computational capabilities. Today, computer simulations have become an excellent way to model and understand social processes. The usefulness of social simulation modeling results as much from the process (problem specification, model development, and model evaluation) as the product (the final model and simulations of social system dynamics). Nevertheless, social simulations need to be a theory-guided enterprise. Its results will often be the development of explanations, rather than the prediction of specific outcomes [2].

Agent-based simulations allow us to develop a new way of thinking, parallel to analytical procedures, based on the emergence of complex behaviors from relatively simple activities [2]. It has great potential to contribute to the discovery of simple social effects when social scientists build very simple models that focus on some small aspects of the social world and discover the consequences of their theories in the "artificial society" they have built.

One of those simple models already built is the well-known Axelrod model for the study of dissemination of culture [8–10]. Axelrod argued that culture "is something people learn from each other," and hence something that evolves through social influence. To study the process of cultural diffusion, Axelrod built a model based on two simple assumptions:

(1) people are more likely to interact with others who share many of their cultural attributes and

(2) these interactions tend to increase the number of cultural attributes they share (thus making them more likely to interact again).

The Axelrod model has been exhaustively implemented to study a great variety of problems: the nonequilibrium phase transition between monocultural and multicultural states [11], the cultural drift driven by noise [12,13], nominal and metric features [14,15], propaganda [16], time evolution dynamics [17], the resistance of a society to the spread of foreign cultural traits [18], finite-size effects [19], the impact of the evolution of the network structure with cultural interaction [20,21], and mobility of social agents [22], among others. The dynamical influence on the society is described in the original Axelrod model using a dyad interaction which, at each time step, only comprises the source and the target of influence in a particular interaction. As discussed by Flache and Macy in Ref. [23], the influence a person feels from the society is a *social* phenomenon that cannot be reduced to the interactions within a dyad given by source-target couple of persons, because the social pressure on the target to adopt an opinion is proportional to the number of people that the target perceives are supporting this opinion. On the contrary, the Axelrod model assumes that the influence is interpersonal (dyadic). We are then in front of two completely different assumptions with respect to the way people interact in a society:

(1) dyadic interaction where it is supposed that two people in a relationship interact in isolation from others and

(2) social influence which is multilateral and involves all network neighbors simultaneously.

In Ref. [23] the authors explained that the central implications of Axelrod's model profoundly change if the dyadic interaction is changed to social influence. The authors showed that the combination of social influence with homophily (the principle that likes attract) solves two important problems which contradict the empirical pattern:

(1) the original Axelrod model predicts cultural diversity in very small societies, but monoculture in larger ones [32] and

(2) when cultural perturbation is present, diversity is obtained only in a very narrow window of noise level and this window decreases with increasing population size [11,12].

On the other hand, one of the extensions of the Axelrod model that has been explored consists of the study of mass media effects over the society [24–27]. Surprisingly, the mass media have been found to be a destabilizing factor which drives the system to a multicultural state [28]. Exceptions are the works [29–31] where different models were introduced.

The aim of this work is to investigate the effects of mass media over the society when the social influence (in the sense of Ref. [23]) is considered. This paper is organized as follows: in Sec. II a brief description of the Axelrod model and its results are explained; in Sec. III we introduce our model, which considers social influence in combination with homophily and mass media. The results are analyzed in Sec. IV and finally some conclusions are outlined in the last section of the paper.

## II. BRIEF DESCRIPTION OF THE AXELROD MODEL

The Axelrod model consists of a population of agents, each one occupying a single node of a square lattice of size $L$ and area $L \times L$. The culture of an agent is described by a vector of $F$ integer variables $\{\sigma_f\}$ called *features* ($f = 1, \ldots, F$). Each feature can assume $q$ different values between zero and $q-1$. These are the possible *traits* allowed per feature. In the original Axelrod model the interaction topology is regular bounded (nontoroidal). Each agent can interact only with its four neighbors (a von Neumann neighborhood) which are the closest (only one step distant from the target agent of influence) without crossing the borders. Initially, individuals are assigned a random culture and the parameter $q$, which defines the possible traits in each cultural dimension, can be seen as a measure of the initial disorder or cultural variety in the system. In the temporal dynamics of the model at each time step $t$, the cultural profile of a randomly selected agent $i$ may be updated through the interaction with a randomly chosen neighbor $j$. The probability of this interaction is proportional to the corresponding overlap of their cultural profiles (the amount of features with identical traits). When interacting, agent $j$ influences agent $i$ causing the latter to adopt $j$'s trait on a feature randomly chosen from those that they do not share. The process outlined above continues until no cultural change can occur. This happens when every pair of neighboring agents has cultures that are either identical or completely different.

In the model, one can explore the aggregate behavior of the system by studying the spatial distribution of the emergent cultural regions: sets of spatially contiguous agents who share an identical vector of culture. Two final possible states are possible: only one cultural region is obtained or multiple cultures are obtained separated by a boundary. These states are called monocultural and multicultural (or global, polarized), respectively. A first-order phase transition is obtained

for $F > 2$ between the monocultural and multicultural states for increasing values of the initial diversity $q$ [10].

The Axelrod model has shown how diversity with local convergence can be obtained. Nevertheless, as mentioned before, there are two results of the model which contradict empirical patterns: (1) the Axelrod model predicts cultural diversity in very small societies, but monoculture in larger ones, and (2) when cultural perturbation is present, diversity is obtained only in a very narrow window of noise level and this window decreases with increasing population size [11,12]. This has been justified as a consequence of the dyadic interaction considered between the agents of the society [23].

## III. MODEL FOR SELF-INCLUDED SOCIAL INFLUENCE WITH MASS MEDIA

We have developed an interaction procedure in the Axelrod model to study the effects of the social influence between agents [23]. At a given time $t$ an agent $i$ is randomly chosen for possible influence. Around any agent, we consider a neighborhood defined by a parameter $p$, which controls how many agents can be visited (in unitary steps) in each direction starting from that agent until the perimeter of the neighborhood is reached. One unit step allows us to move only in the north-south or east-west direction. When $p=1$ the neighborhood obtained is therefore the well-known von Neumann neighborhood $N_1$, which only includes the four closer neighbors of an agent ($N_1 = \{j_1, j_2, j_3, j_4\}$). In our case, $p$ can take higher values to define bigger neighborhoods. In general, for a given value of $p$ it is obtained $N_p = \{j_1, \ldots, j_\alpha\}$, where $\alpha = \alpha(p) = 2p(p+1)$. This gives four neighbors ($\alpha = 4$) for one-step neighborhood ($p=1$), 12 neighbors ($\alpha = 12$) for two-step neighborhood ($p=2$), and so forth.

The social influence is defined as follows: when an agent $i$ is selected for possible influence, all of its neighbors (according to the value of $p$) are included in the set of influences $I_i$ with probability $p_{ij_\alpha} = O_{ij_\alpha}/F$, where $O_{ij_\alpha}$ is the cultural overlap between agents $i$ and $j_\alpha$. The set $I_i$ obtained by this procedure becomes a set of traits' influences over agent $i$'s traits. Our approach is similar, but not exactly equal, to that proposed in Ref. [23]. In our model we have decided to include also agent $i$ into the set $I_i$ with probability $p_{ii} = 1$. This allows a self-reflection of agent $i$ about its own traits when comparing with the traits of its neighbors. Then, each agent is aware of its own traits when being influenced by its neighbors and its own traits count when deciding whether or not its corresponding value is changed to a different one.

Figure 1 illustrates the social influence procedure. In the example shown, the agent $i$ selected has cultural traits $(1,1,1)$ and the neighborhood is formed by the one-step neighbors (von Neumann neighborhood). We have also supposed that all the four neighbors have entered into the set of influences $I_i$. In general, neighbors are included according to the probabilities $p_{ij}$, computed for each neighbor. The last agent to be included on the set $I_i$ is an agent shared by all the agents on the network. Without loss of generality this superagent $s$ is set up with culture zero in all of its trait values and never changes them. This global agent represents a mass media

Agent $i$      Set of Influence $\mathbf{I}_i$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \longrightarrow \quad \mathbf{I}_i = \left\{ \begin{matrix} j_1 & j_2 & j_3 & j_4 & i & s \\ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, & \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, & \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, & \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{matrix} \right\}$$

$p_{ij} = 1/3$          $p_{ii} = 1$   $p_{is}$

FIG. 1. Social interaction between agent $i$ and its neighbors. Each neighbor is included with probability $p_{ij_\alpha}$ into the set of influences $I_i$. The agent $i$ is also included with probability $p_{ii}=1$. The agent $s$, included with probability $p_{is}$ according to Eq. (1), represents a superagent which acts over the whole society.

influence which acts over the whole society. Its inclusion in this model follows the same rules explained in Ref. [31], according to which the probability of interaction between this superagent and an agent $i$ of the society is given by

$$p_{is} = \frac{O_{is} + \epsilon}{F + \epsilon}, \tag{1}$$

where $O_{is}$ is the overlap between the agent $i$ and the super-agent $s$. The parameter $\epsilon$ $(>0)$ can take real values and it is a measure of the mass media strength to act over all agents in the society. Then, the superagent $s$ has always a nonzero probability $[\min(p_{is})=\epsilon/(F+\epsilon)]$ to be included in the set of influences of the agent $i$. This probability is proportional to the strength parameter $\epsilon$ and its nonzero value means clever mass media which design its actions as to have always something in common with the target of influence.

Once the agent $i$ is randomly selected and the current influence set $I_i$ is established (including $i$ and $s$), an agent $i$'s feature is selected at random with a trait value different from the corresponding value of at least one agent of the set $I_i$ in the same feature. The decision of which trait value agent $i$ adopts (if any change takes place) is done in such a way that agent $i$ imitates (or copy) the most common trait in the set $I_i$ for the feature selected. If, for example, in the situation depicted in Fig. 1 the feature selected on agent $i$ were the second (that in the middle), this trait value ($\sigma_{i,2}=1$) will be changed to $\sigma_{i,2}=0$ due to the highest frequency of this value in the set $I_i$. If the feature selected were the third (from top to down), the value $\sigma_{i,3}=1$ will remain due to the highest frequency of this value on the set $I_i$. The last possible case presented in Fig. 1 occurs if the first feature were selected. In this case, on the set of influences $I_i$, we have the same frequency of appearance for the values 0 and 1. In this situation, one of the two values is chosen at random and, then, the trait value $\sigma_{i,1}=1$ in agent $i$ could remain with a probability of 1/2 or be changed to $\sigma_{i,1}=0$ with the same probability. This is another difference with the model discussed in Ref. [23], where there is no change if the trait value selected shares the maximum frequency with others. In summary, the trait value of agent $i$ is changed to the value of the corresponding trait with the highest frequency of appearance on $I_i$. If there were more than one, it is changed to one of them with equal probability. The trait value of agent $i$ does not change if it is the one with the highest frequency of appearance on $I_i$.

At the same time, we have also included *interaction* errors

as well as *copying* errors, just as in Ref. [23]. The interaction error relaxes the previous deterministic procedure used in former works of Axelrod model when deciding the possible interaction between two agents (dyadic interaction). Both copying and interaction errors can randomly alter the outcome of an interaction event. The interaction error acts over the selection procedure as follows: if the normal procedure of selecting an agent $j$ for being included in the set of influences $I_i$ results in its inclusion, then with probability $r'$ the neighbor $j$ is removed from $I_i$. If the neighbor has not been selected into the influence set, then with probability $r'$ the agent $j$ will be included into $I_i$. This error creates the possibility that cultural influence occurs across the boundary of two disconnected cultural regions if a neighbor with zero overlap is included in the influence set. This error can also reduce the social pressure against adopting different trait values when a cultural identical neighbor is excluded from the set of influences and increases the possibility for the target agent $i$ to adopt trait values from a completely different culture. On the other hand, the copying error acts after agent $i$ has adopted (or not) a different trait value and the new trait value has been already set up. In this case, the corresponding trait value adopted by agent $i$ is changed to a new randomly selected value with probability $r$. Notice that the new value randomly generated could be that one the agent $i$ already has. In general, both interaction and copying errors are conceptually different, but for simplicity we have used here the same value $r=r'$. As seen on the description of both types of noise errors, they operate in the process of changing the culture information of a target agent. This process includes two steps: the first one when considering the neighbors to be taken into account for influence (where the interaction error activates) and the second step when the target agent adopts a specific trait value (where the copying error activates). Then, both errors are only present when a target agent of influence is active. As a first approximation we have not included a "temperature" error where agents change their trait values at random with certain rate. Additionally we have implemented periodic boundary conditions (toroidal society) to avoid boundary effects.

In the case of dyadic interaction and in the absence of noise, when analyzing the possible final absorbing states, it is simpler to establish it by checking when each agent has full or null overlap with each one of its neighbors. In the case of social influence the problem is more involved. However, it is also possible to establish some technical conditions to be checked to see if an agent is active (i.e., can interact with its neighbors according to the dynamical rules established in the model) or not taking into consideration the set of influences $I_i$. When all the agents of the society are inactive, then an absorbing state is obtained.

In the present study we have also implemented the procedure developed in Ref. [32] where a list of active agents is built. In this way, instead of randomly selecting agents of the society in each time step, the agents are randomly chosen directly from this list. This procedure strongly increases the efficiency of the dynamical evolution of the system and it allows us to save computational time. Therefore, when the system is initialized by randomly assigning different cultures to each agent of the society, the first list of active agents is

built. Next, at each time step, when the influence is established and an agent of the society changes its cultural value, the list of active agents is updated analyzing the agent itself and all its neighbors to check which of them are now active. The dynamical iteration keeps on until the length of the list of active agents is reduced to zero. It is worth noticing that in our case (where social influence is established), some runs did not settle into a final well-defined absorbing state. In these cases the list of active agents reduces to one element and each time this agent changes its cultural values and becomes inactive, one of its neighbors becomes active. This propagation goes on indefinitely. We have neglected these cases from our calculations and we have only considered runs which finish in a precise well-defined final absorbing state. Moreover, when temperature-noise type is included, it is not possible in general to define a final absorbing state and the dynamical evolution of the system is stopped by defining some criteria related to a given definition for the stationary state attained by the system. In our case, as explained before, the copying and interaction errors are only present on those agents which are active at each time step. This allows the system to reach well-defined final absorbing states.

## IV. RESULTS

We have implemented different computational experiments to study the mass media effects in the Axelrod model when the social influence is the mechanism at work for agent interaction. As there is no qualitative difference on the dynamical behavior of the system for $F>2$, we have set $F=3$ in our study, a network size with $L=50$ (2500 agents unless otherwise stated), and averages have been taken over 200 different initial random configurations.

As mentioned previously, the original Axelrod model shows a first-order phase transition (for $F>2$) for a critical value of the initial diversity $q_c$. For low values of $q$ ($<q_c$) the low initial diversity allows the system to reach a final global state, but for $q>q_c$ the high initial diversity reduces the cultural convergence and the system reaches a final polarized state. Nevertheless, as it will be shown, when the mass media are included with social interaction the system has a non-monotonic dependence with the values of $q$.

In Fig. 2 we represent the normalized value of $S$ (the amount of agents in the biggest culture domain when the system reaches the final absorbing state) as a function of the initial diversity $q$ with no noise ($r=0$). Figure 2(a) corresponds to calculations with one-step neighborhood ($p=1$) while in Fig. 2(b) the calculation was done for $p=2$. This means that the amount of neighbors of each agent in Fig. 2(a) is only four plus the superagent while in the case of Fig. 2(b) each agent has now 12 neighbors beside the superagent. Different values of the superagent strength $\epsilon$ are included. Also included in dotted line is the parameter $\delta$ which is the amount of information of the superagent on the society. This parameter is obtained counting along all of agents and their corresponding traits if the information is equal to that of the superagent in the same feature. The final count is divided by $F$ to scale the value in a way that can be shown in the same scale as the parameter $S$.
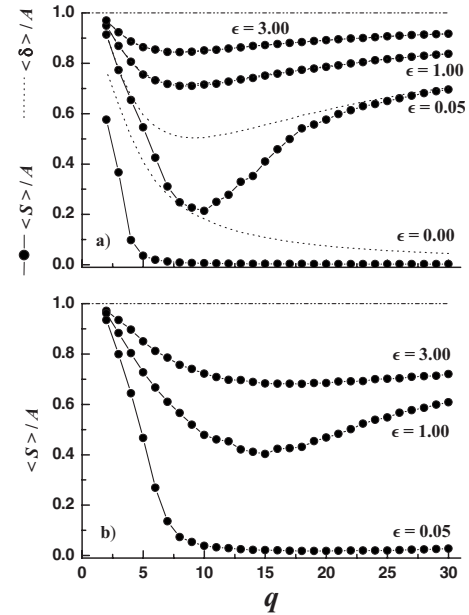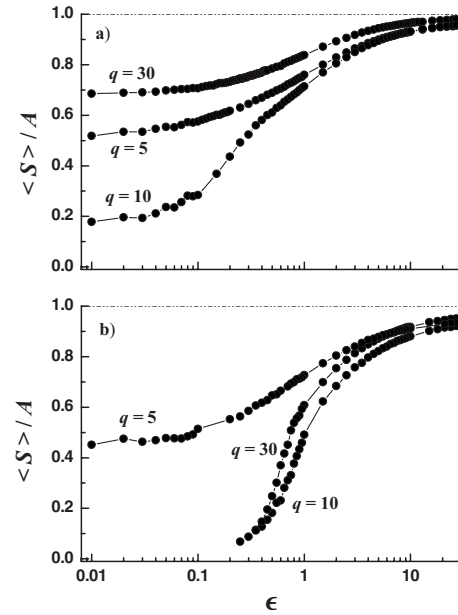


FIG. 2. Final absorbing state in the Axelrod model with social influence as a function of the initial diversity $q$ for different values of the mass media strength $\epsilon$. The dot-symbol line is the fraction of the biggest culture while dotted line is the fraction of mass media information on the society [only in (a)]. The parameter $A$ is the area of the system ($A=L^2$). (a) Calculation done including one-step neighbors (von Neumann neighborhood). (b) Calculation done including two-step neighbors. The calculation is done with the number of features $F=3$, lattice size $L=50$, and averages are taken over 200 different random initial conditions.

It can be seen that for low values of $q$ the system reaches a close-monocultural state for any value of the strength $\epsilon$. These are rather trivial cases because these extremely low values of $q$ mean a very low initial diversity of the system and a final close-monocultural state is then expected.

A monocultural state is also induced for high enough values of $q$ but the system is now more sensitive to the values of $\epsilon$ than in the region of low $q$ values. In this case, higher values of $\epsilon$ induce a stronger monocultural final state given by higher values of $\langle S \rangle$. To explain this result, we note that higher values of $q$ mean that there is initially a higher degree of cultural diversity on the society and this is reflected in sets of influences $I$ with low number of neighbors, i.e., each set of influences $I_i$ will be frequently composed by the own agent $i$ and by the superagent $s$, and with low probability by the neighbors since they probably do not share any of their trait values with agent $i$. The probability of the superagent to be included in the set of influences $I$ increases when increasing the value of $\epsilon$. For higher $q$ values, in cases where $I_i$ is formed by agent $i$ and superagent $s$, the last one will be able to introduce its own value on agent $i$ with a probability of 0.5. The iteration of this process in time drives the system to a close-monocultural state.

A more interesting situation occurs for middle values of $q$ where it is observed a minimum of the $\langle S \rangle/A$ values as a function of $q$, more pronounced for $\epsilon=0.05$ and $q=10$ in Fig. 2(a) and $\epsilon=1.00$ and $q=15$ in Fig. 2(b). For values of $q$ close to the corresponding minimum, the initial diversity is such

that besides agents $i$ and $s$, some of the agent's neighbors are also included in the set of influences $I$. Then the interacting dynamics involves higher cultural diversity on the set of influences $I_i$ and the superagent fails to induce a strong monocultural final state.

It is interesting to note that when comparing Figs. 2(a) and 2(b) it can be seen that the increase in the number of neighbors included in the social interaction decreases the size of the biggest culture in the absorbing state and then a more pronounced multicultural state is reached for the same values of $q$ and $\epsilon$. The value of $q$ where the minimum of $S$ is attained also increases. This is a consequence of a direct competition between the higher diversity on the set $I_i$ and the homogenized influence of the superagent. In Fig. 2(b), the amount of neighbors to be analyzed for inclusion on the set $I_i$ is 12, which is three times the case in Fig. 2(a) with only four. Then with $p=2$, at any value of $q$ there will be, with higher probability, bigger diversity of trait values than in the case of $p=1$ and, therefore, a weakening of the homogenizing effects of the superagent is expected. This decreases the possibility of the superagent to drive the system to a monocultural absorbing state because the probability for a trait value of the superagent to appear with the highest frequency on the set of influences is lower, and lower values of $S$ are obtained. Then, when considering more neighbors in the social interaction the higher local diversity reinforces the final multicultural state, even with the presence of mass media.

On the other hand, when comparing the dotted line for $\langle S \rangle$ with the dotted line for $\langle \delta \rangle$ in Fig. 2(a) it can be seen that they both coincide for all values of $q$ when $\epsilon$ has higher values ($\epsilon=1.00$ and $\epsilon=3.00$). It means that the information of the superagent is present only in the biggest culture of the society. Nevertheless, for small values of $\epsilon$, the two curves coincide for higher values of $q$, but the values of $\langle \delta \rangle$ are higher than the values of $\langle S \rangle$ for middle values of $q$. Then, in this situation although the final state does not correspond to a strong monoculture, the information of the superagent on the society includes the bigger culture and also agents which belong to smaller culture sizes.

Another important parameter of the system is the mass media strength $\epsilon$. Its values could control the homogenization of the society through global influence. References [25,26] report the counterintuitive result that increasing mass media strength produces polarization. Nevertheless, as explained in Ref. [31], the model in Refs. [25,26] and the one discussed here are not compatible. Then, it is of further interest to study qualitatively the dependence of the final absorbing state with respect to increasing values of mass media strength as it is defined in this work.

Figure 3 shows a second computational experiment. Here, we have included the values of the same parameter $\langle S \rangle / A$, but now as a function of the superagent strength $\epsilon$ for three different values of $q$. In this case, noise has also been neglected. One- and two-step neighborhoods are included in Figs. 3(a) and 3(b), respectively. For the three values of $q$ shown, a monoculture is strongly induced as $\epsilon$ increases due to the constant presence of the superagent on the set of influences. The order of the curves according to the value of $q$ is in correspondence with the order of the curves in Fig. 2 and the values of $\epsilon$. As $\epsilon$ increases the superagent becomes a



FIG. 3. Final absorbing state in the Axelrod model with social influence as a function of the mass media strength $\epsilon$ for different values of the number of traits (per feature) $q$. (a) Calculation done including one-step neighbors (von Neumann neighborhood). (b) Calculation done including two-step neighbors. The parameters used for these calculations are the same as in Fig. 2.

factor of "normalization" making constant its own values in time along the dynamical evolution of the system. Even for $q=10$, when the set of influences includes with high probability some of the neighbors of the target agent and the superagent traits' values are not on the majority frequency of appearance, the latter succeed in inducing a monoculture as the strength $\epsilon$ increases. In particular, for $\epsilon \geq 1$, an almost monocultural state with $\langle S \rangle / A \gtrsim 0.7$ is already induced, i.e., at least 70% of the society belongs to the biggest culture for all values of $q$ included. For $\epsilon=1$, we have $\min(p_{is})=0.25$ (this is obtained considering null overlap between the superagent $s$ and an agent $i$) but the superagent can induce the 70% of the society to belong to the bigger culture with its own information (according to the value of the parameter $\delta$ shown in Fig. 2), for any initial diversity $q$. The maximum value of $\epsilon$ included in Fig. 3 is $\epsilon=27$, which gives $\min(p_{is})=0.90$. Furthermore, when comparing Figs. 3(a) and 3(b) the same results are obtained as when comparing Figs. 2(a) and 2(b). In this case the inclusion of two-step neighbors in the social influence dynamics of the system strongly decreases the value of $S$ for the cases $q=10$ and 30.

In order to get additional insights on the system dynamics, we next consider the same model but including noise. Here, we want to test the results obtained in Ref. [23] related to the stability of the system to different noise levels when social influence is at work. We are also interested in studying the interplay between different noise rates and different mass media strengths. Higher noise rates could polarize the system but strong mass media could cancel out this effect. We have conducted different calculations and in Fig. 4 it is reported the dependence of the parameter $\langle S \rangle / A$ with respect to different noise levels for several values of $q$, $\epsilon$, and $L$. In Figs.
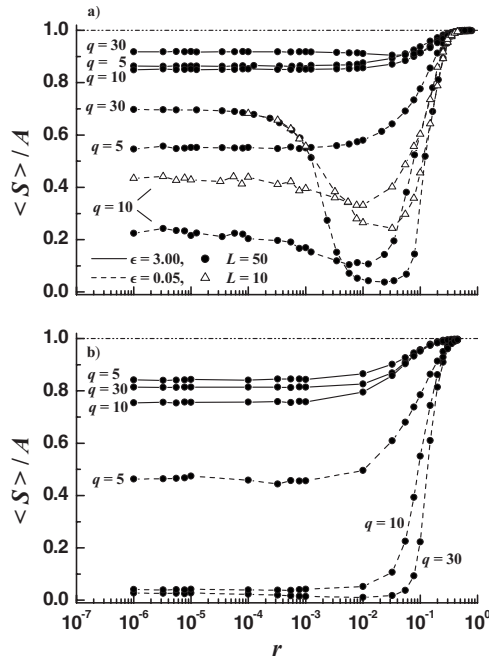
FIG. 4. Dependence of the final absorbing state with respect to different noise levels and different values of the initial diversity $q$. The solid line includes the case with mass media strength $\epsilon = 3.00$ while dashed lines are the cases with $\epsilon = 0.05$. Black circles report the final absorbing states calculated with a network size of $L = 50$, while white triangles are the case with $L = 10$. (a) Calculation done including one-step neighbors (von Neumann neighborhood). (b) Calculation done including two-step neighbors.

$4$(a) and $4$(b) we have considered one- and two-step neighborhoods, respectively. The noise included ranges between 0 and 0.45. As can be seen, social influence makes absorbing states more stable to a bigger range of noise levels than dyadic interaction, as already reported in Ref. [23]. In general there is no qualitative change for at least three orders of magnitude for all the values of $q$, $\epsilon$, and $L$ used. For a higher strength of mass media [$\epsilon = 3.00$, $\min(p_{is}) = 0.5$] the final absorbing state remains almost monocultural for four orders of noise level, and finally for noise values higher than 0.1 the system is driven to a full monocultural state reached at $r = 0.45$ approximately. The situation is different for low values of superagent strength ($\epsilon = 0.05$). In this case the final absorbing state is also stable to different noise levels, but only for three orders of magnitude. In Fig. $4$(a) it can be seen that for $q = 5$ and noise levels higher than $r = 0.01$, the system is driven to a monoculture. When increasing the noise level for $q = 10$ and 30 (for $\epsilon = 0.05$), there is first a reinforcement of the multicultural state, given by decreasing values of $\langle S \rangle / A$ (strongly induced for $q = 30$). Moreover, for even higher values of $r$ a monoculture is induced. All the calculations shown in black circles have been done with a network size of $L = 50$. In order to study if the stability of the final absorbing state to noise is robust with respect to the network size, we have also explored the case $L = 10$. Quantitative differences arise only for $q = 10$ and 30, for $\epsilon = 0.05$ [white triangles in Fig. $4$(a)]. In general it is observed that the final absorbing state is stable to the same range of noise independent of the network size. The difference obtained, consisting

of the higher values of $\langle S \rangle / A$ for $q = 10$, and the higher minimum for $q = 30$ at $r = 0.05$ (both for $\epsilon = 0.05$) are due to the lower size of the network ($L = 10$). There are no quantitative differences for the other parameters due to the rather trivial case of $q = 5$ (for any value of $\epsilon > 0$) and for the high value of $\epsilon$ used in the solid lines.

In Fig. $4$(b) we have included the same results in Fig. $4$(a) calculated with $L = 50$, but now considering two-step neighbors. The results are qualitatively the same for $\epsilon = 3.00$ (no matter what the values of $q$ are) and for $q = 5$ and $\epsilon = 0.05$. For the case of $\epsilon = 0.05$ and $q = 10$ and 30 the system ends in a multicultural state for almost all the range of noise levels. Nevertheless, for sufficiently high noise level the system is abruptly driven to a monocultural state, as in all the others cases.

Here, we advance an analysis for possible explanation of the results obtained on this experiment. There are three parameters involved: the initial cultural diversity $q$, the strength of the superagent $\epsilon$, and the noise level $r$; and two different error mechanisms: the copying and including errors. For low values of $\epsilon$ the superagent will appear on the set of influences $I_i$ with low frequency. This frequency increases for increasing $\epsilon$. For low values of $q$ the majority of the neighbors of the target agents will be included on $I_i$ because of the low cultural diversity, while the opposite occurs for large enough values of $q$. Nevertheless, as can be seen from Fig. 4, this complex interplay does not have an important impact on the dynamics of the system along a wide range of noise levels and the value of $\langle S \rangle$ remains almost constant. When $r \gtrsim 0.1$ the copying error becomes the dominant mechanism because independent of the cultural trait an agent has copied from its neighbors, the copying error changes it to any one randomly selected and, as it was already said, it deletes the boundaries between different cultural regions. It is also of importance that the error mechanism stops when the agent becomes inactive. Hence, the social influence is the mechanism which allows an agent to be active or inactive and it is also the mechanism which switches on or off the copying error. Then, as the copying error connects two completely different cultures the homogenization is favored and agents become inactive (and noise stops) when its culture is completely equal to its neighbors. Therefore the final monoculture obtained at $r = 0.45$.

At lower values of noise a nonmonotonic behavior is obtained for $q = 10$ and 30 when the strength $\epsilon$ has low values ($\epsilon = 0.05$ in this case). The value of $\langle S \rangle$ first decreases for increasing noise reinforcing the multicultural state. This effect is much pronounced at $q = 30$. In these cases, the initial diversity makes the set $I_i$ formed only by the target agent and some of its neighbors. More neighbors will be present on the set of influences for $q = 10$ than for $q = 30$. For increasing noise the including error makes the set $I_i$ populated by both agents from cultures which share some traits with the target agent and also with agent neighbors which do not. The superagent will also be included. This interplay seems to produce strong local convergence and it drives the system to a multicultural state for a range of noise between 0.01 and 0.1. Finally, as already noted, the copying error drives the system to a monoculture state for even higher values of $r$.

Finally, we study the influence of the mass media in the culture-area relation when the interaction on the society is
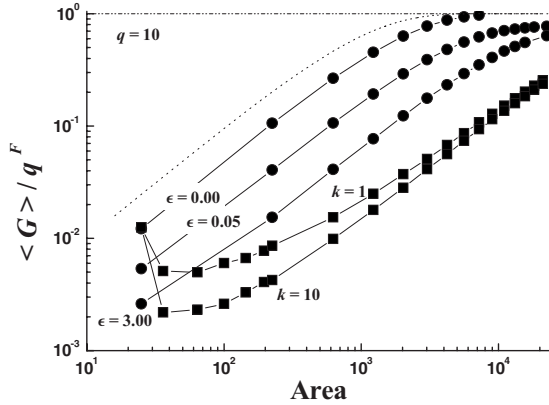
FIG. 5. Culture-area relationship for initial diversity $q=10$ and different values of mass media strength $\epsilon$. Circles include the calculations for the corresponding value of $\epsilon$ indicated with labels. Squares represent the calculations done according to the relation $\epsilon = k(L-5)$, with the value of $k$ indicated with a label. Number of features $F=3$, lattice size $L=50$, and 200 different random initial conditions were used. The area is obtained according to $A=L^2$.

social. An extensive examination of the culture-area relation on the Axelrod model with dyadic interaction was done in Ref. [32]. The authors obtained a nonmonotonic behavior for the culture-area relation for $q$ values below the critical value where the transition occurs ($q < q_c$), while for $q > q_c$ the number of cultures $G$ first increases in a power-law dependence as $G \sim A^x$ with $x=1$ and then gradually flattens when the area becomes on the order of the maximum number of cultures $q^F$. The parameter $G$ counts all cultures expressed on the society, even if it is composed of only one agent. In the limit $L \to \infty$ there are only two possible outcomes: for $q < q_c$ a single culture dominates in an ordered regime, while $G \to q^F$ and all the cultures are represented in the network in a full disordered regime. The authors proved the transition between these two regimes to be discontinuous because $G$ jumps from 1 to $q^F$ at $q=q_c$. Contrarily, when the model is changed including instead the dyadic interaction social influence [23], the problematic result of the Axelrod model which produces a multicultural final state only for small societies [32] is overcome and only multicultural final states are obtained. In case the mass media are included according to the model used in Refs. [25–27] which take the interaction between agents as dyadic, it was proven in Ref. [28] that even for a vanishingly small probability of the mass media to interact with agents is sufficient to destabilize the homogeneous regime for very large lattice sizes, contrary to the usual common sense.

Here, we have calculated the amount of different cultures present in the final absorbing state indicated by the parameter $G$. We have followed the definition of Ref. [32] where all the different culture configurations obtained are counted without paying attention to simple connected regions. As there are $q^F$ different possible cultures, the parameter $G/q^F$ is normalized and its maximum value ($G/q^F=1$) means a completely multicultural state while low values are related to monocultural final states.

In Fig. 5 the final cultural diversity $G$ is included with circles for a fixed value of the mass media strength $\epsilon$. It can be seen that at $\epsilon=0$ the amount of cultures obtained in the final absorbing state increases for increasing value of the area and reaches the maximum multicultural state for sufficiently high value of $A$. The calculation done with any other value of $q$ gives the same dependence of the area which includes the same slope and only a parallel shift. The other curves were then not included for simplicity. We have then qualitatively reproduced the results obtained in Ref. [23]. In our case, a calculation of the slope yields $x=0.42 \pm 0.02$, which is different from that reported in Ref. [32]. We think that the deviation is due to the difference that arises when dyadic interaction or social influence is included.

When the superagent is included the values of $G$ are lower for higher strength $\epsilon$ at the same value of the area, but the slope of the curve is the same as when the superagent is not present. The superagent also prevents the system to reach the full multicultural state ($\langle G \rangle/q^F=1$) for higher values of the area. This can be seen on the curve for $\epsilon=0.05$, which saturates at values of $A > 10^4$. The same seems to occur with $\epsilon=3.00$. Calculations taking in consideration two-step neighbors were also carried out with the same qualitative results (including slope and saturation). Only a shift to smaller values of $G$ was obtained for the cases of $\epsilon=0.00$ and 0.05, but the shift finally disappears for $\epsilon=3.00$, meaning that for strong enough mass media the relative importance of the amount of neighbors included in the set of influences is weak (at least for the value of $q=10$).

Furthermore, on the dynamics of the system there are two parameters which compete to produce opposite effects for increasing values of each one. For increasing area of the network the system tends to reach a multicultural state while an increasing value of the superagent strength $\epsilon$ pushes the system to a more cultural homogeneous absorbing state. In order to study the relative weight that these parameters have over the system, we have also calculated the final absorbing state when both area $A$ and strength $\epsilon$ increase. To quantitatively accomplish this purpose we have established a linear dependence of the strength $\epsilon$ to the area given by the following relation:

$$\epsilon = k(A - 5), \qquad (2)$$

where the parameter $k$ allows us to control the increasing rate of the mass media strength $\epsilon$ when the area $A$ increases. Then, in this case a larger value of the area also guarantees a larger value of the mass media strength.

In Fig. 5 we report, with square symbols, the results for $k=1$ and 10. The values of the diversity $G$ obtained are higher for $k=1$, than for $k=10$, due to the lower rate of increase in the strength $\epsilon$, and consequently a more multicultural final state is induced. Nevertheless, both curves increase for increasing area, which means that the area of the network has more weight than the strength $\epsilon$ on the dynamics of the system. The increments of the mass media strength only has important effects on the probability $p_{is}$ for low values of $\epsilon$, where $p_{is}$ appreciably changes. In our case, the maximum overlap between the superagent and any agent of the network is $\max(O_{is})=F=3$. Then, for $\epsilon/F \gg 1$ [which occurs rapidly for increasing value of $A$ in Eq. (2)] it is obtained that $p_{is} \to 1$ [$p_{is}$ saturates for increasing $\epsilon$ as seen in Eq. (1)] and the

relative increase in the probability $p_{is}$ is canceled out with the effects of the increasing area. This explains also why the curves for $k=1$ and 10 tend to the same values as the area is increased. A potential dependence of $\epsilon$ on the area will yield the same result for low values of the area.

Then, when social interaction is present according to the present model, increasing network size always drives the system to a multicultural state, while increasing superagent strength prevents the system to reach the maximum possible of cultural configurations. Additionally, the saturation value of $G$ seems independent of the value of the strength of the mass media, as far as our calculations have shown.

We have also included in dotted line in Fig. 5 the analytical expression reported in Ref. [32],

$$\langle G \rangle = q^F (1 - e^{-A/q^F}), \tag{3}$$

which is the average number of cultures in the totally disordered configuration where $A = L^2$ agents are randomly assigned with one of the $q^F$ available cultures. The expression is valid for $A$ and $q^F$ very large. As can be seen, the prediction from Eq. (3) overestimates the values for the cultural diversity as a function of the area. The results are more different the higher the values of the superagent strength $\epsilon$ are since it is a factor that decreases the cultural differences between neighboring agents.

## V. CONCLUSIONS

Summing up, we have numerically studied an Axelrod model in which two different mechanisms are at work. The first takes into account that agents are prone to change their state according to a sort of social influence given by a (variable) neighborhood made up of their nearest (both in space and in cultural similarity) neighbors. Two differences with respect to the model reported in Ref. [23] were included in our model: the first one is the inclusion of the own target agent on the set of influences and the second one allows the possibility of cultural changes when there are more than one culture with maximum frequency on the set of influences. The first difference favors the polarized final state while the second one favors the evolution to final global states.

At variance with [23], the dynamics of the agents is also ruled by a mass media effect, represented in this case by an agent that keeps fixed its cultural traits all the time. The dynamics of the system is such that at low and high values of initial social diversity $q$, a monocultural state is attained, with a stronger dependency on the mass media strength ($\epsilon$) at large $q$ values. The reason behind the formation of these monocultural states is the varying size of the set of neighbors with social influence: the higher is the diversity of initial cultural traits, the less is the number of agents in the neighborhood of an agent. Admittedly, the more interesting behavior is for intermediate values of $q$, where the system dynamics attains a minimum in the size of the biggest monocultural cluster. At those values of $q$ the initial diversity is such that the mass media have to compete with a larger number of neighbors of the agent and thus its information is not necessary on the majority. Then, in this case the mass media fail to drive the system to a homogeneous cultural state. Neverthe-

less, the increase in the mass media strength for a fixed network size always reinforces the monocultural state.

On the contrary, the increase in the network size was proven to be a parameter which strongly drives the system to a polarized society where all the possible cultural configurations are present. The amount of final number of cultures also follows a power-law dependence when the social influence is at work, but the exponent value found here is lower than that reported for the case of the original Axelrod model with dyadic interaction. When mass media are present these only succeed in preventing the full multicultural state for increasing size of the society and the amount of cultures obtained saturates. Moreover, the saturation value was found to be independent of the mass media strength in our calculations.

We have also studied the model with noise representing errors in copying traits and in the formation of the social influence interactions. These types of errors are only present as part of the process which defines the culture values of an agent when it is active. This allows the system to reach well-defined final absorbing states and correspondingly a higher precision in the description of the noise effects. The results show that social influence makes the system dynamics more stable against the presence of noise, and that the latter only has a marginal influence on the general qualitative picture obtained without any errors. It is however worth stressing that the noise included has in general a positive effect in the formation of monocultural final states, giving rise to global societies for large enough values of it, independent of the other parameters ruling the size of the social influence and the strength of the mass media effect. Nevertheless, the fact that we have not distinguished between the two error mechanisms in this study does not allow us to better understand the specific role each one plays on the dynamics of the system and further studies are needed where these two types of errors were considered as independent. We also stress the need to study the same system when a temperaturelike error is present. This is due to the possible strong implication that such a noise might have on the final absorbing state of the system, as now inactive agents may become actives again affecting the stability of the absorbing state. We hypothesize that it might be possible that higher enough temperature-noise rates produce final polarized states.

We also point out the need to conduct more detailed studies exploring the implications of the social influence with respect to the dyadic one. The introduction of different models which better study the inter-relations between the dyadic interaction and social influence on a social system could be of importance to further elucidate the robustness of the results found here. This work is currently in progress.

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