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TOPOLOGICAL VERSUS DYNAMICAL ROBUSTNESS IN A LEXICAL NETWORK

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Semantic memory is the cognitive system devoted to the storage of conceptual knowledge. Empirical semantic networks constructed from adult generated free word association data represent a good map for this system. Concepts, represented as words, link each other with a certain weight representing the strength of their relation. Everyday experience shows that word search and retrieval processes, which can be assimilated to traversals on a semantic network, provide fluent and coherent speech, i.e. are efficient and robust. Brain pathologies, such as Alzheimer's disease or schizophrenia, severely damage neural structures and associated capacities. Thus, semantic networks must also undergo disruption, but the question is how long cognitive processes, which depend on the underlying structures, can operate before collapsing. Interestingly, we find that degradation of the original structure has a dramatic impact on the topology of semantic network, whereas the dynamics upon it evidence much higher resilience. We define this problem in the framework of percolation theory.

Keywords: Information retrieval; complex networks; weighted percolation; semantic impairment.

1. Introduction

Modern network theory [Boccaletti *et al.*, 2006] is progressively contributing to our understanding of cognition. Although it has not yet penetrated cognitive science as, for example, social science or biology, these last years have witnessed an increase in the works that use statistical physics methodology to gain insight into cognitive phenomena. Language growth [Dorogovtsev & Mendes, 2001]; child language development [Steyvers & Tenenbaum, 2005; Hills *et al.*, 2009, 2010]; category formation and search processes [Borge-Holthoefer & Arenas, 2010a] or verbal fluency [Goñi *et al.*, 2009, 2010] stand as good examples of such strategy, see [Borge-Holthoefer & Arenas, 2010b] for a review. Significantly, all these works are devoted to language and cognitive processes around it. The reason for this is that semantic memory, the cognitive system where conceptual knowledge is stored, can be suitably represented as a network, where nodes represent words and links between pairs of nodes stand for word–word relationships. Representation of language as a network has a long tradition in cognitive science [Quillian, 1967; Collins & Quillian, 1969; Collins & Loftus, 1975], and complex network theory represents a methodological update of those proposals, boosted by massive empirical data availability.

Network modeling of language is mostly devoted to normal, healthy performance. However, understanding how aging and disease affect proficiency in language production and comprehension is a great concern in the field. In this paper, we present theoretical results which analyze how deterioration is related to performance. To this end, we first place the problem of semantic breakdown in the context of error and attack tolerance [Albert *et al.*, 2000; Cohen *et al.*, 2000; Schwartz *et al.*, 2002]. There exists a rich literature regarding network robustness, breakdown and final disintegration in the cognitive field [Achard *et al.*, 2006; Kaiser *et al.*, 2007; Alstott *et al.*, 2009]. However, as we shall discuss later, the previous approach is not satisfactory. We then define an appropriate framework to model aging and pathologies in the cognitive context. Several studies from cognitive neuroscience report a progressive loss of structural and functional connectivity in brain networks in patients compared with control subjects [Stam *et al.*, 2007; Liu *et al.*, 2008; Supekar *et al.*, 2008; Stam *et al.*, 2009]. It is on the tracks of this fact that cognitive deterioration is modeled. After that, and beyond the expected results, i.e. performance is impoverished in the frame of illness, we study the efficiency of a particular dynamics on a degenerated structure. That is, in the line of [Duch & Arenas, 2007], we focus on the relationship between topological robustness and dynamical robustness, by comparing when the network is first physically split or dynamically collapsed. This comparison is made under different assumptions, the main conclusion is that semantic memory is strongly shaped by statistical biases and this elicits a more efficient and robust performance.

2. Topology: Free Association Norms

Association graphs are networks in which vertices denote words, whereas links represent association relations as observed in cognitive-linguistic experiments. Such graphs are considered the most relevant from a psycholinguistic point of view and can be taken as a proxy to the actual structure of semantic memory, because of their comprehensive character: they may for example represent semantic relationships — the shared semantic context of *car* and *road* — but also functional or causal relationships — as in *fire* and *smoke* — among others. Association norms reflect in some aggregate way important regularities in language, and are strongly predictive of adult performance in different kinds of language processing tasks. According to the hypothesis that association is one of the principles of memory organization, the question that has to be addressed is which network topologies support an efficient organization in terms of time and space complexity.

The best known Free Association data set in English are University of South Florida Free Association Norms (FA from now on; [Nelson *et al.*, 1998]). Nelson *et al.* [1998] produced these norms by asking over 6000 adult participants to write down the first word (*target*) that came to their mind when confronted with a *cue* (word presented to the subject). The experiment was performed using more than 5000 cues. Associative strength is the frequency of coincidence between subjects for each pair of words. As an example, words *mice* and *cheese* are neighbors in this database, because a large fraction of the subjects related this target to this cue. The empirically obtained network is a directed and weighted graph. Weights represent the frequency of association in the sample, and their distribution is highly heterogeneous. These same data also exist in Spanish [Callejas *et al.*, 2003; Fernández *et al.*, 2004], German [Melinger & Weber, 2006] or French [Ferrand & Alario, 1998].

Although most of the literature on small-world and scale-free networks has focused on the undirected, unweighted version of FA, we believe that the directed network is clearly a more natural representation of word associations. Thus asymmetry is preserved throughout this work.

3. Dynamics: Random Inheritance Model

The Random Inheritance Model (RIM) [Borge-Holthoefer & Arenas, 2010a] explores whether it is possible to disentangle similarity relationships from general word association network (FA) by a naïve cognitive navigation on top of it. Unlike association strength, which is often asymmetric and phenomenological (in the sense that associate words might be so for very different reasons), similarity relationships express the degree of overlap in the meaning of two words. Such relation is then symmetric. More specifically, two words are considered semantically similar if they share features. Note that similarity lies at the base of category formation.

The process can be schematized as uncorrelated *random walks* from node to node that propagate an *inheritance mechanism* among words, converging to a feature vectors network, see Fig. 1. Besides the degree of success when compared to empirical data (see below), the most appealing feature of RIM is two-fold: (i) the fact that it implements an intuitive idea — we repeatedly navigate a semantic network to produce or understand meaningful utterances, and an aggregate of these explorations can plausibly resemble random walks; (ii) a dynamics governed by uncorrelated random walks admits

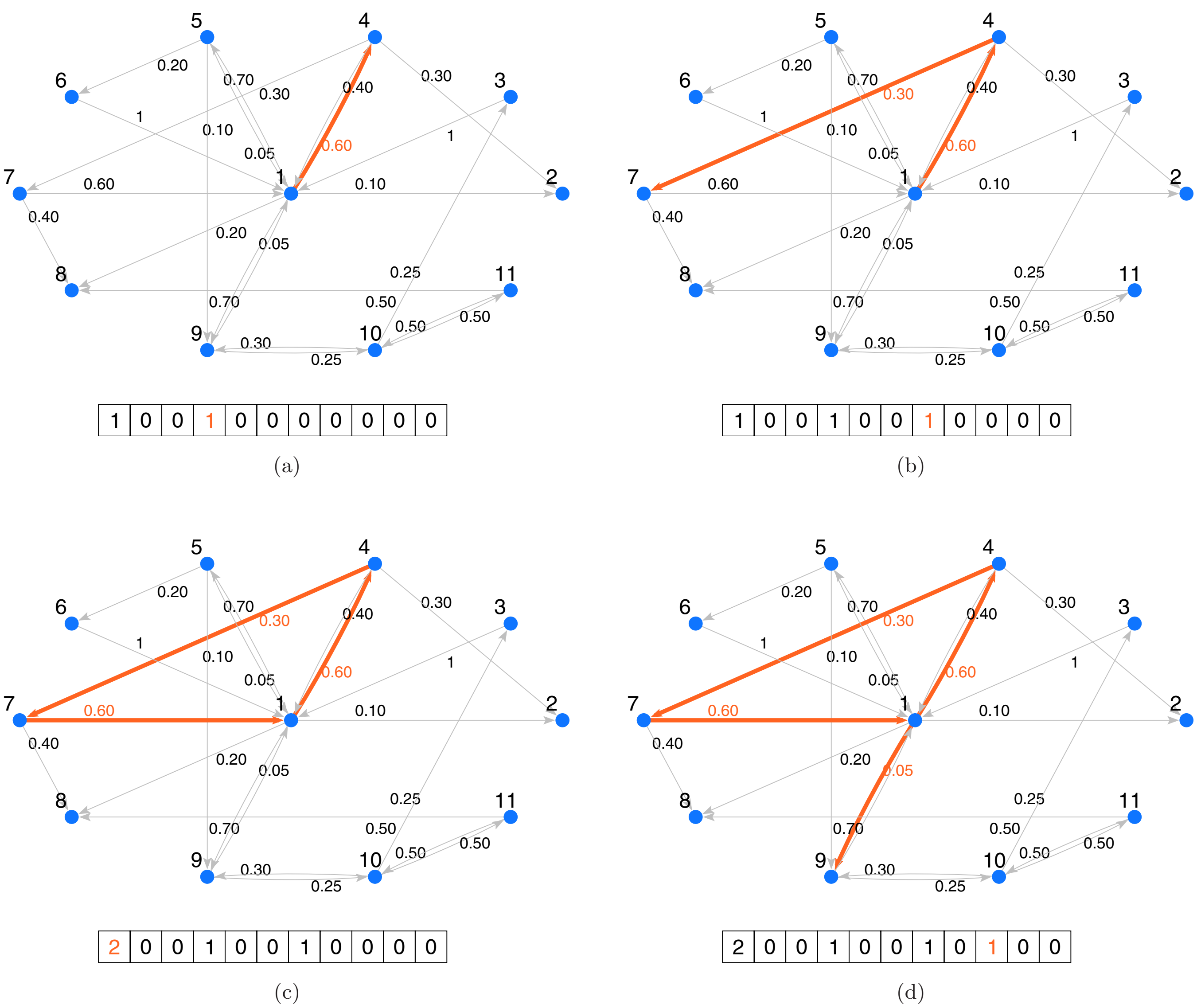

Fig. 1. In RIM, the visits of a random walker starting at node i trigger the inheritance mechanism, which modifies the features vector of a node i. In the figure, a random walk of 4 steps (a)–(d) changes the vector of node 1.

a clear formalization, allowing for an analytical understanding of the emergence of semantic similarity among words.

Regarding (ii), RIM can be algebraically described in terms of Markov chains. To this end, we must define the transition probability of the FA network. The elements of FA (a_{ij}) correspond to frequency of association, these weights can be normalized and the adjacency matrix corresponding to the FA network becomes a transition probability matrix. The transition probability matrix P is thus defined as:

$$P_{ij} = \frac{a_{ij}}{\sum_j a_{ij}}. \tag{1}$$

As the original matrix, this one is also asymmetric. Once the matrix P is constructed, the random walkers of different lengths are simply represented by powers of P. In practice, this means that if we perform random walks of length S, after averaging over many realizations we will converge to the transition matrix P^S, every element $(P^S)_{ij}$ represents the probability of reaching j, from i, in S steps. The inheritance process corresponds, in this scenario, to a change of basis, from the orthogonal basis of the N-dimensional space, to the new basis in the space of transitions T:

$$T = \lim_{S\to\infty} \sum_{i=0}^{S} P^i = (I - P)^{-1}. \tag{2}$$

In practice, we are rather interested in metastable or quasi-stationary states of the Markov process (finite S) [Meyer, 1989], i.e. the length of the random walkers is limited. The summation in Eq. (2) converges, in terms of the matrix 1-norm, very fast, limiting the dependence on indirect associative strengths [Nelson & Zhang, 2000]. Although computations were done for several values of S, $S = 4$ is enough to reach quasi-stationary states in T, thus results for RIM in this work are expressed for $S = 4$ from now on. Finally, the goal of this dynamics is to quantify semantic similarity, which is calculated as the cosine of the vectors in the new space, given by the scalar product of the matrix and its transpose, $FS = TT^{\dagger}$.

The results obtained by RIM show macro-statistical coincidences (functional form of the distributions and descriptors) between real semantic similarity data and the synthetic obtained network (FS). The model also yields significant success at the microscopic level, i.e. it is able to reproduce to a large extent empirical relationships, see Table 1 and [Borge-Holthoefer & Arenas, 2010a] for details.

Table 1. An illustrative example of RIM's predictive capacity, when comparing closest neighbors to empirical data. The ten most similar concepts to the word ROOSTER are listed, sorted in decreasing order.

ROOSTER	
Empirical	RIM
Chicken	**Chicken**
Goose	**Turkey**
Pigeon	Crow
Sparrow	Robin
Penguin	**Sparrow**
Pelican	**Bluejay**
Bluejay	**Pigeon**
Dove	**Pelican**
Hawk	**Goose**
Turkey	**Hawk**

In conclusion, after a random walker-based dynamics each node (word) obtains a new neighborhood, different from the one in FA, made of other words which hold similarity relations with it. Now the question is whether these inferred semantic neighborhoods are stable, i.e. the dynamics is robust, when semantic memory is under the influence of some pathology, i.e. when the substrate (FA) supporting the dynamics undergoes degradation.

4. Topological and Dynamical Robustness

In the previous section we have proposed a mechanism that drives the emergence of category structure. Now we turn to the characteristics of both the original topology and RIM dynamics under error. Literature on error and attack tolerance in complex networks [Albert *et al.*, 2000; Cohen *et al.*, 2000; Schwartz *et al.*, 2002; Boguña & Serrano, 2005; Serrano & Boguña, 2006] typically model deterioration in two ways: error as the failure (removal) of randomly chosen nodes/edges, and attack as the removal of important nodes/edges ("importance" can be quantified by some descriptor, be it high connectivity, high betweenness, etc.). Using this approach, one typically monitors a suitable network characteristic that signals the moment in which physical disintegration of the structure takes place. As for RIM's dynamics, performance of a word i

($match_i$) under error or attack is measured as the proportion of words that remain similar to i in the impoverished structure, compared to the original results. This implies that, each time a node is removed, RIM is applied on the distorted structure and current similarity neighborhoods are compared to the original ones, for each node. The quantity "match" is a global average over the N elements of the network, and of course its value is 1 when no node has yet been removed.

In Fig. 2 we use this approach by progressively removing nodes. This has been done in three ways: randomly choosing the failing node (failure), choosing it in terms of highest vertex betweenness (maximum betweenness attack), or eliminating the node with highest ω_{in} (maximum in-strength attack).

At least two conclusions can be drawn from Fig. 2. In the first place, the relative size of the giant component $N_{\text{giant}}/N_{\text{net}}$ decays in a similar way to those reported in the literature for scale-free networks [Albert *et al.*, 2000], i.e. the structure is robust against failures but attacks hinder the integrity of the topology much before, approximately at $f = 0.75$. More interestingly, the RIM dynamics are not as resilient as the structure, and collapses long before the topology is actually disintegrated in the three cases (failure and both attack strategies). That is, before the critical point is reached, the dynamics' performance is much more deteriorated than the topology for any given fraction of removed nodes, f.

Though informative, we now wonder whether failure or attack correctly grasp the way in which pathologies or aging affects a cognitive structure such as semantic memory. Empirical evidence in the neuroscience literature report on a general decay of the neural structure supporting cognition [Stam *et al.*, 2007; Liu *et al.*, 2008; Stam *et al.*, 2009]. Then, realistic modeling demands a different way to approach this problem. Here, we redefine error in the context of cognitive systems. In this framework, it is more useful to consider error in terms of aging or disease, where the whole topology simultaneously decays in some way. By doing so, we capture the degrading behavior of aging and/or disease, which differs from attack (there is no selective action) and from error (which affects only one node/edge at a time). For the sake of clarity, we refer to error in the cognitive framework as degradation.

Degradation assumes that links are increasingly damaged. At a given threshold τ, every link (i, j) in FA with a $\omega_{ij} \le \tau$ is removed. The surviving links are normalized [Eq. (1)] to preserve a probabilistic interpretation of the structure. This process is performed with values $0 \le \tau \le 1$. As in the case of failure and attack, for each value τ, we monitor

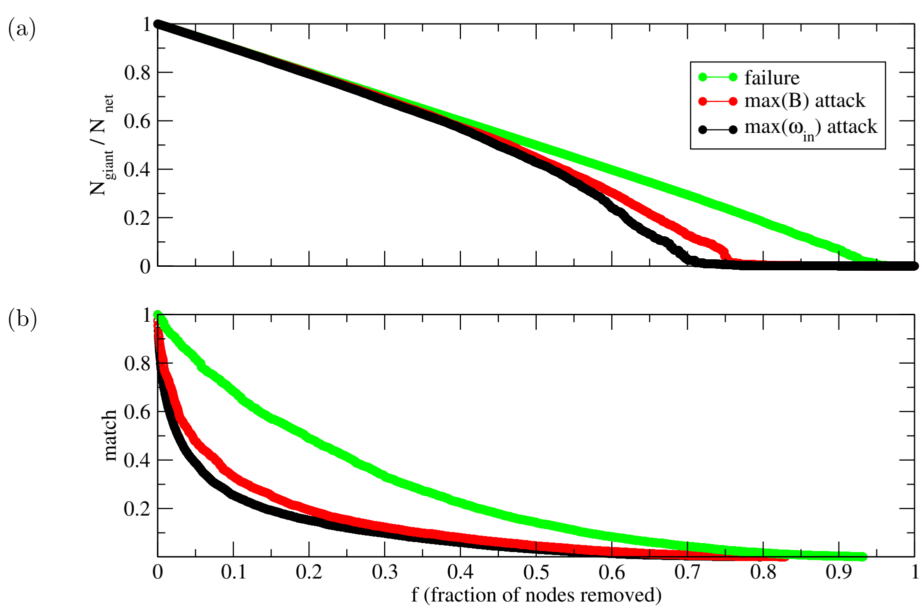

Fig. 2. (a) Topological deterioration (relative size of the giant component) of FA as a function of f, the fraction of nodes removed from the network. In green, results for error (random failure of nodes); in red, results for attack to vertex betweenness: the nodes with highest B are removed first. Finally, attack to higher in-strength appears in black. (b) RIM's resilience for the same strategies.

both topological and dynamical properties of the resulting network (the size of the giant component of the degraded structure is measured; and RIM is used to find a similarity matrix on the degraded structure, and the result is compared to the nondegraded RIM, i.e. RIM's result at $\tau = 0$ — "match" axis in figures).

4.1. *Degradation on the original structure*

Figure 3 shows the results for both topological deterioration (a) and dynamical resilience (b). Focusing on black circles (which correspond to degradation of the FA network), the behavior of RIM's dynamics appears to be very sensitive to degradation even at very low values of τ. This suggests that lexical impairment can appear at early stages of semantic memory disease degradation. Interestingly, however, RIM's degradation is much slower than the topological one. At $\tau \approx 0.3$, FA structure is already disintegrated, whereas RIM can still recover as much as 25% of its original content. RIM's results do not vanish up to $\tau \approx 0.6$. Such result indicates that fundamental cognitive capacities such as word–word similarity inference and category formation are substantially resilient to structural impoverishment.

4.2. *Degradation on the null model I*

Results in black circles (degradation of the FA network) from Fig. 3 raise the question of which topological aspect of the dynamics' substrate provides for good performance in a disrupted topology. To answer this question, we propose to build appropriate null models. The idea is that, by changing the topology that supports the dynamics, we can gain insight on which properties of the original structure provide long-lasting performance. In particular, we devise two null models, both preserve the degree sequence and directions. In the first place, we consider the network FA in which weights are ignored. This implies that each node has the same in- and out-degree distribution, but outgoing links are weighted uniformly, that is $\omega_{ij} = 1/k_{\text{out}}$. The topological resilience of this modified FA network is represented as red circles in the top panel of Fig. 3. The percolation point has moved to the left if compared to the original results, i.e. the network is structurally weaker. This result is not surprising, given that τ affects in the first place weaker links; thus, in an unweighted FA, this means that (i) nodes with higher degree k_{out} have the lowest weights, $1/k_{\text{out}}$; (ii) since weights are distributed uniformly for each node, when $\tau = 1/k_i$ *all* outgoing links are removed at once. In this sense, this

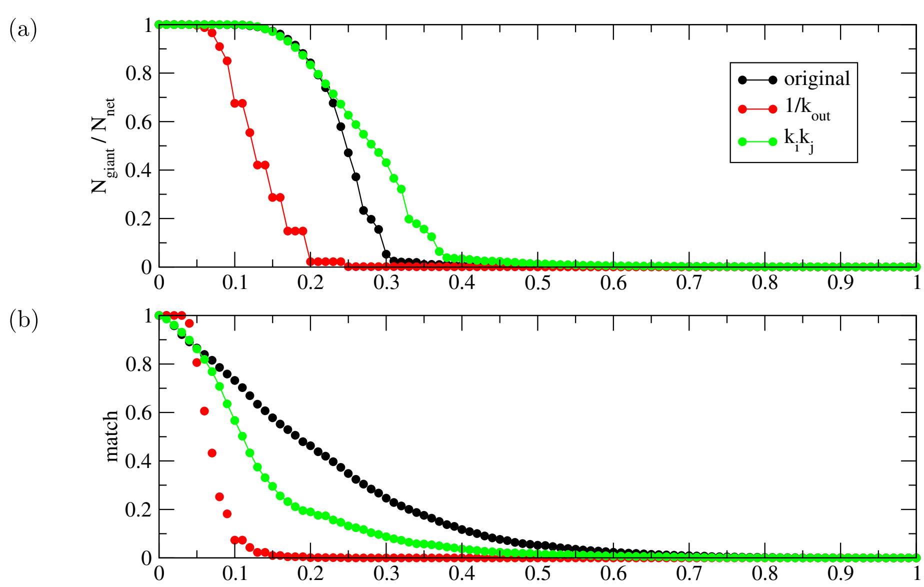

Fig. 3. (a) Topological deterioration (relative size of the giant component) of FA as a function of τ. In black, results for the original FA structure. The same process of degradation has been applied to an unweighted version of FA understood here as a plausible null model, in red. In green, results for a second null model, which sets the weights of links in FA as $k_i^{\text{out}} k_j^{\text{in}}$. (b) RIM's resilience for the same structures.

null model is equivalent to a $k_{\max}$-attack. From the dynamical point of view, it is apparent also that the structure cannot support category formation and similarity inference for a long time: RIM's results collapse even before the topology reaches the topological percolation point.

4.3. *Degradation on the null model II*

Although the unweighted null model is not an "aggressive" one (it preserves many of the features of the original network, such as degree distribution $P(k)$, average degree $\langle k\rangle$, average clustering coefficient C, etc.), we may devise one in which, furthermore, weight heterogeneity is still present. This can be done by assigning a weight to out-links proportional to the out-degree k_i^{out} of the source node i and the in-degree of the node j receiving that link, k_j^{in}. Then, $\omega_{ij} = k_i^{\mathrm{out}} k_j^{\mathrm{in}}$. The weights quantified in this fashion are normalized, to replicate the probabilistic interpretation of the original links in FA and in the previous null model. As it is apparent from the green circles in Fig. 3 (upper panel), the $k_i^{\mathrm{out}} k_j^{\mathrm{in}}$-weight configuration yields a more resilient structure from a topological point of view, the percolation point is displaced to the right if compared with the original FA. This is so because, contrary to the previous case, nodes with high degree are favored, in such a way that both their in- and out-weights are high. Then the threshold parameter τ does not affect hubs until a late degradation stage, the structure is not severely fragmented until $\tau \approx 0.4$. However, RIM decays faster than the original FA counterpart (Fig. 3, lower panel). Although the value of "match" vanishes approximately at the same time as for the original substrate, dynamic deterioration for early τ values is more rapid.

5. Conclusions

Up to this point, we have introduced free association norms, and in particular FA, as a plausible representation of the structure of semantic knowledge. Also, we have presented RIM as a proxy of real cognitive dynamics to extract the category structure backbone from a general semantic relations context (FA). Once this information is available we study how dynamics reacts when confronted with failure and attack, in the first place; and then with progressive degradation of the topological structure. To this end, we follow the line of percolation theory in complex networks with some modifications. Results indicate that linguistic performance is severely affected by semantic memory degradation, on the other hand, such performance is still significantly effective beyond topological disintegration.

Our study concludes, furthermore, that the specific distribution of weights in the lexical network plays a key role in the resilience both of the topology and the dynamics. Perturbing this weight distribution dramatically changes the capacity of the structure to hold performance dynamics on it. The particular value of these weights is just a consequence of contextual diversity (statistical biases of language use), which can soundly be identified as the origin of categorization and the maintainer of semantic integrity. On the other hand, this work raises some questions of interest. From a physical point of view, the new approach to structural damage demands an analytical treatment, in order to predict the topological response to weighted degradation. In this line, a reconsideration of current knowledge on percolation theory is necessary.

From the standpoint of neuroscience and psycholinguistics, attention should focus on how physical (neurological) and cognitive degradation are related. Also, it has been reported that pathologies sometimes selectively affect linguistic performance [Moss & Tyler, 2000; Caramazza & Mahon, 2003]. Then some kind of "selective degradation" should be implemented and studied, given the modular structure that lexical networks display [Borge-Holthoefer & Arenas, 2010a, 2010b]. Finally, other variables can be taken into account; for instance, at the moment a node is disconnected from the network, its "cognitive load" (the semantic meaning it bears) must be assumed by the remaining connected structure. In this way, degradation could be in interplay with node-breaking avalanches [Moreno *et al.*, 2002, 2003], which could explain not only cognitive dysfunction (inexact or impoverished semantic capacities) but also system inefficiency (general performance slowing).

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