International Journal of Bifurcation and Chaos

Effects of traffic properties and degree heterogeneity in flow fluctuations on complex networks

This paper is for the Special Issue of "Dynamics and Processes on Complex Networks"

Sandro Meloni
Institute for Biocomputation and Physics of Complex Systems (BIFI), University of Zaragoza, 50009 Zaragoza, Spain

Department of Informatics and Automation, University of Rome "Roma Tre", Via della Vasca Navale, 79 00146, Rome, Italy

Jesús Gómez-Gardeñes
Department of Condensed Matter Physics, University of Zaragoza, 50009 Zaragoza, Spain

Institute for Biocomputation and Physics of Complex Systems (BIFI), University of Zaragoza, 50009 Zaragoza, Spain

Vito Latora
School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, London E1 4NS, United Kingdom.

Dipartimento di Fisica e Astronomia, Università di Catania, and INFN, Via S. Sofia 64, 95123 Catania, Italy

Laboratorio sui Sistemi Complessi, Scuola Superiore di Catania, Via San Nullo 5/i, 95123 Catania, Italy

Yamir Moreno
Institute for Biocomputation and Physics of Complex Systems (BIFI), University of Zaragoza, 50009 Zaragoza, Spain

Department of Theoretical Physics, University of Zaragoza, 50009 Zaragoza, Spain

Received (to be inserted by publisher)

Communication networks are nowadays crucial in our lives and the study of its traffic features yields important advantages. In both network and traffic design the understanding of the relationship between the traffic on a node and its fluctuations plays a key role. In this paper we investigate the relationship between the mean traffic flow experienced by a node and its standard deviation via numerical simulations and real data analysis. In particular, we show the great influence that the degree heterogeneity of real communication systems has on the patterns of flow fluctuations observed across complex communication networks. To this end, we derive an analytical law connecting the standard deviation of flows and their mean values, we prove it via extensive numerical simulations and by means of a realistic internet traffic simulator software: *NS-3*. We also show that our results are robust under different assumptions regarding: network topology, routing strategy and packets injection distributions.

Keywords: Flow Fluctuations, Traffic Dynamics, Complex Networks

1. Introduction

Communication networks are of crucial importance in our interconnected world as nowadays they are the basic backbone for most of human, commercial and social activities. For these reasons the study of communication networks is receiving an intense and increasing attention in different fields, from computer science to physics and mathematics [Pastor-Satorras & Vespignani, 2004; Albert & Barabási, 2002; Pastor-Satorras *et al.*, 2001]. Specifically, in the last years two topics became of high relevance for the physics community. On one side, the study of the structure of real complex networks showed the intimate relationship between their topological patterns [Newman, 2003] and their dynamical processes taking place on them [Boccaletti *et al.*, 2006; Dorogovtsev *et al.*, 2008]. For what communication networks concerns, their structural patterns [Serrano *et al.*, 2005] turn out to be the key feature behind their robustness and efficiency in the spreading of information [Albert *et al.*, 2000; Crucitti & *et al.*, 2004; Sreenivasan *et al.*, 2007]. On the other side, in the recent years a prominent role has been taken by the study of theoretical models aimed at describing the complex traffic dynamics generated on communication networks and to improve their performances [Ohira & Sawatari, 1998; Solé & Valverde, 2001; Guimerá2 *et al.*, 2002; Tadic *et al.*, 2007; Rosato *et al.*, 2008; De Martino *et al.*, 2009]. In this line, many efforts have been invested to find strategies to avoid system congestion so to enlarge the traffic capacity of communication networks [Guimerá *et al.*, 2002; Echenique *et al.*, 2004, 2005; Yan *et al.*, 2006; Wang & Zhou , 2007; Meloni & Gómez-Gardeñes, 2010]. Additionally, as usually large communication networks such as the Internet operate in the under-congested (free flow) state the study of the traffic conditions in this regime is of primary importance [Jacobson & Karels, 1988; Kurose & Ross, 2008]. In this latter context, one of the main factors affecting the dynamical conditions of a communication network are the fluctuations in traffic flows [Eisler *et al.* , 2008; Argollo de Menezes *et al.*, 2004; Duch & Arenas, 2006; Meloni *et al.*, 2008]. Fluctuations in traffic levels can have different causes: from circadian-like behaviors (such as day/night or week days cycles) to exceptional events such as malfunctioning and unusual traffic levels. An inadequate response to unexpected flow fluctuations could lead to damages and failures from the single node level to, in extreme cases, affect the entire communication network [Motter & Lai, 2002; Crucitti1 & *et al.*, 2004; Buldyrev *et al.*, 2010].

The malfunctioning due to unexpected flow fluctuations in communication systems illustrate the importance of quantifying of the standard deviation σ of the mean traffic levels $\langle f \rangle$ for the design and maintenance of communication networks. In the last years the study of the relationship between σ and $\langle f \rangle$ has received a lot of attention in the literature. In [Argollo de Menezes *et al.*, 2004] the authors derived a simple scaling relation $\sigma \sim \langle f \rangle^{\alpha}$ with the existence of only two universality classes: $\alpha = 1/2$ and $\alpha = 1$. These results were questioned in [Duch & Arenas, 2006] where the authors assumed the same scaling form ($\sigma \sim \langle f \rangle^{\alpha}$) but, via queue theory arguments and numerical simulations, they demonstrate that the exponent α varies in the entire range $1/2 \leq \alpha \leq 1$. More recently, we introduced [Meloni *et al.*, 2008] an analytical model based on random walk theory: the so-called random diffusion (RD) model. Thanks to the RD model we were able to demonstrate that the exponent α really varies between 1/2 and 1 and, more importantly, that the simple power-law scaling, previously supposed, should be abandoned for a more complex form that depends on the interplay of three factors: the fluctuations in the packets generation, the degree of the node and the length of the time window used to collect the statistics. Although we proved our findings via numerical simulations and real data analysis some questions remained open.

In this paper we employ a realistic communication networks simulator (*NS-3*) to investigate the effects of nodes degree in the scaling between the mean flow and its fluctuations on nodes. Moreover we also show the robustness of the prediction of the RD model over a series of different realistic hypothesis regarding namely: the routing scheme utilized for traffic forwarding, varying between the shortest path and a traffic aware protocol, the traffic conditions from free flow to congested state, different traffic creation rate distributions and, finally, the structure of the underlying network. In all these cases we show that, despite of its simplicity, the RD model can be used to predict the standard deviation of the mean flux over nodes and, thus, applied to design and control real networks. The article is organized as follows. In section 2 we recall the main features of the random diffusion model already presented in [Meloni *et al.*, 2008]. Section 3 is devoted to the presentation of the numerical model we use for studying of the robustness of the results derived from the RD model under more realistic conditions. In section 4 we use *NS-3* to investigate the effects of degree heterogeneity. Finally, in section 5 we draw our conclusions.

2. Analytical model

In this section we present the RD model as a simple analytical framework to study effects of network structure and traffic variations on the mean flows in a communication network. As shown in our previous work [Meloni *et al.*, 2008] the RD model allows to calculate the average number of packets that go through a node and its fluctuations in case that packets diffuse randomly. In the RD model we start considering a network of N nodes and K links on which w packets can diffuse. Packets are defined as non interacting random walkers, so we assume that each node is able to process all the packets it receives (no queues are used) and, once a packet reaches a node of degree k, it will jump with probability $1/k$ to one of the node's neighbors. In this scenario, by means of random walk theory arguments, we obtain an expression for the mean number of walkers λ_i on node i once the stationary regime has been reached. λ_i read as [Noh & Rieger, 2004; Gómez-Gardeñes & V. Latora, 2008]:

$$\lambda_i(w) = \frac{k_i}{2K} w \; . \tag{1}$$

To collect the mean flux on a node we discretize the observation time T into different time windows (each of length M). We consider a time window as the minimum period needed to measure the number of walkers that go through a node. Specifically, we define the flux on a node as the number of packets traveling through the node in M time units. In this period we measure the average number of packets $\langle f_i \rangle$ processed by node i, together with its standard deviation σ_i.

To take into account possible external fluctuations in the number of packets arriving to the system, we consider two possible situations. In the first one we assume no external fluctuations in the packets generated by the system, meaning that, the number of packets in the network is constant over the entire observation period T and thus we have $w = W$. In the second situation we assume differences in the creation rate of the packets from one time window to the other. We guarantee that the mean value of the number of walkers in the network in a time window is equal to w by assuming that the probability $F(w)$ of having w walkers on the network in a window of length M is equally distributed in the range $[W - \delta, W + \delta]$, *i.e.*,

$$F(w) = \frac{1}{2\delta + 1} \; , \tag{2}$$

with $1 \leq \delta \leq W$. Within this framework, we can start our analysis to find an expression for $\langle f_i \rangle$ the average number of packets reaching node i.

We first calculate $P_i(n)$ as the probability that, after M time steps, n packets flowed through node i. If we consider the case in which the packets creation rate is constant for the whole observation period ($w = W$) and the fact that packets are not interacting, we can define the arrival of walkers on a node as a Poisson process. Thus, we can write an expression for the mean number of packets (the average flux) at a node i after a period of M time units as $\langle f_i \rangle = \lambda_i(w)M$, and also the probability of having n packets as:

$$P_i(n) = \mathrm{e}^{-\lambda_i(w)M} \frac{(\lambda_i(w)M)^n}{n!} \; , \tag{3}$$

with a mean value $\langle f_i \rangle = \lambda_i(w)M$ and the same value for the variance, σ^2, leading to $\sigma = \sqrt{\langle f_i \rangle} = \sqrt{\lambda_i(w)M}$. This clearly leads the scaling exponent α to be equal to $1/2$.

If we extend our analysis to the more general case in which the number walkers w is distributed according to Eq. (2), the probability $P_i(n)$ reads as:

$$P_i(n) = \sum_{j=0}^{j=2\delta} \frac{\mathrm{e}^{-\frac{k_i}{2K}(W-\delta+j)M}}{2\delta+1} \frac{[\frac{k_i}{2K}(W-\delta+j)M]^n}{n!} \, . \tag{4}$$

Now, to obtain the mean flux and the fluctuations we can calculate the first two moments of $P_i(n)$ as:

$$\langle f_i \rangle = \sum_{n=0}^{\infty} n P_i(n) = \frac{k_i W M}{2K} \, , \tag{5}$$

$$\langle f_i^2 \rangle = \sum_{n=0}^{\infty} n^2 P_i(n) = \langle f_i \rangle^2 (1 + \frac{\delta^2}{W^2}) + \langle f_i \rangle \, . \tag{6}$$

At this point, recalling that $\sigma_i^2 = \langle f_i^2 \rangle - \langle f_i \rangle^2$, we can write down the relationship between the standard deviation and the mean flux $\langle f_i \rangle$ as

$$\sigma_i^2 = \langle f_i \rangle \left(1 + \langle f_i \rangle \frac{\delta^2}{W^2} \right) \, . \tag{7}$$

The previous formula, although derived from a very simple model, presents some interesting features that can also be found in real world systems. First of all it shows that the relation between σ_i and $\langle f_i \rangle$ mainly depends from three parameters each one related to the three main features of the model: the external arrivals to the system, the time scale of the diffusion process and the structure of the network. The corresponding parameters are: *(i)* δ (the fluctuations in the packets creation rate from one time window to the other); *(ii)* M (the length of the time window over which the mean flows are collected); and *(iii)* k_i (the degree of the node). A deeper analysis of eq. 7 shows that the previous claims about an universal scaling exponent α should be abandoned as we can obtain a continuous range of α values by varying the three parameters of the model. In particular, only if the interplay between the three quantities δ, M and k_i is such that:

$$\frac{k_i M \delta^2}{2KW} \ll 1 \, , \tag{8}$$

expression (7) reduces to a power-law scaling $\sigma \sim \langle f \rangle^\alpha$ with exponent $\alpha = 1/2$. Otherwise, whenever the ratio $\frac{k_i M \delta^2}{2KW}$ is not negligible anymore, the exponent α approaches 1. It's also important to notice that the last expression has a quadratic dependency on the fluctuations δ and only a linear one with M and k_i. For this reason small changes in δ produce large effects on the scaling exponent α while large variations in M and k_i are needed to obtain the same effects on α.

Fig. 1 shows the solution of the RD model for three different cases: *(i)* the value of the noise δ is fixed and solutions for different values of M are obtained, *(ii)* the opposite case in which M is fixed and one solves the model by varying δ and *(iii)* both δ and M are fixed (in this case we show the solution of the model for higher values of k_i). Panel (a) represents the first case in which we fixed the ratio $\delta/W = 0.1$ and the length of the time windows used to measure the flow of packets through different nodes is varied. In this case the scaling $\alpha = 1/2$ only holds for small values of M, while α approaches 1 as M is increased. In this latter case also the more elusive effect of nodes degree k_i is highlighted as for $M = 10^2$ the value of α increases for increasing values of the degree. In panel (b) we depict the influence of the noise level for a fixed time window length ($M = 10$). Our results show that, when δ is small enough so that the number of packets in the network is almost fixed from one time window to another, $\alpha \simeq 1/2$. On the contrary, larger values of δ leads to α values closer to 1. To better highlight the elusive effect of nodes degree on α in panel (c) we present the solution of the model for a fixed time window ($M = 1000$) and ratio $\delta/W = 0.1$ varying k_i from 1 up to $5 \cdot 10^3$. Also in this case we can see that all the values between $0.5 \le \alpha \le 1$ are obtained for different values of k_i.

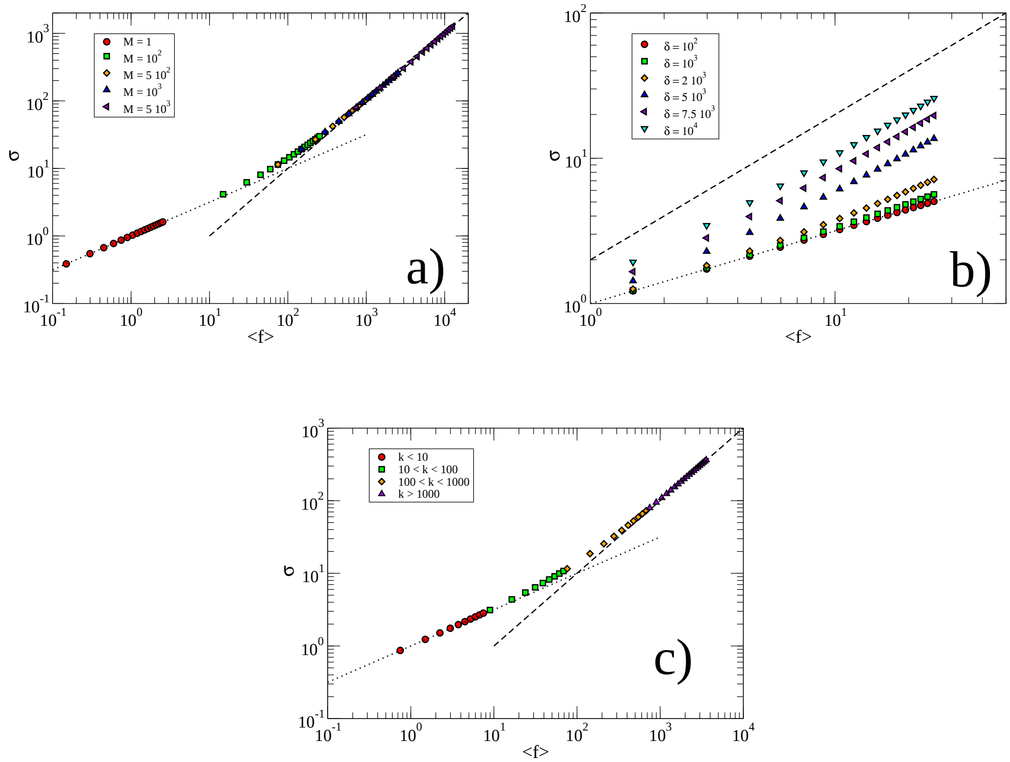

Fig. 1. Standard deviation of the mean flow σ as a function of $\langle f \rangle$ in the RD model with various parameter values. Panel (a): $\delta = 10^3$ and $W = 10^4$. Panel (b), W has the same value while M has been fixed to 10. Points correspond to the solution of Eq. (7) for different values of k_i $(1 \ldots 18)$. Panel (c) $\delta = 5$, $W = 50$ and a time window $M = 10$, in this case k_i varies $1 \leq k_i \leq 5 \cdot 10^3$. The total number of links is $K = 33500$. Dashed lines are guides to the eyes and correspond to $\sigma \sim \langle f \rangle^\alpha$, with $\alpha = 1/2$ (lower curves) and $\alpha = 1$ (upper curves).

3. Numerical results

In this section we show that the relation of eq. 7, although defined in the simplified context of the RD model, holds also in more complex and realistic scenarios. Here we introduce a class of numerical models able to reproduce some of the traffic features usually found on real communication networks.

In our traffic model each node represents a router with an infinite size buffer that contains packets waiting to be forwarded. Buffers are managed with a First-In-First-Out (FIFO) policy. Time is discretized in time-steps T; at each time-step, p new packets are introduced in the system with randomly chosen origins and destinations. Packets routing is based on a shortest path criterion. In other words, each packet is diverted in such a way that the distance d_{ij}, measured as the number of nodes one needs to pass by between node i and node j, is minimized. This strategy will be called *standard* protocol.

In the RD model we assume that agents moves randomly from one node to the others. In our previous work [Meloni *et al.*, 2008] we showed that the predictions of the RD models hold for the *standard* routing protocol. Here, in order to study the robustness of the theoretical prediction with respect to the routing scheme, we also introduce a traffic aware strategy [Echenique *et al.*, 2005]. Specifically, a routing policy based on the shortest path between two nodes neglects the queue in overloaded nodes which makes the process slower as the queue lengths become larger. That is, it may be more efficient to divert a packet through a larger but less congested path. Let us hence assume that a node l is holding a packet that should be sent to a node j and define an effective distance d^i_{eff} from a neighboring node i of l to the destination j as:

$$d^i_{eff} = h_d d_i + (1 - h_d) c_i \tag{9}$$

where d_i is the topological distance between node i and destination j, c_i measures the congestion of node i as the number of packets in i's queue and $h_d \in [0,1]$ is a free parameter. Note that for $h_d = 1$ we recover the *standard* protocol while for $h_d = 0$ packets run randomly between less congested nodes.

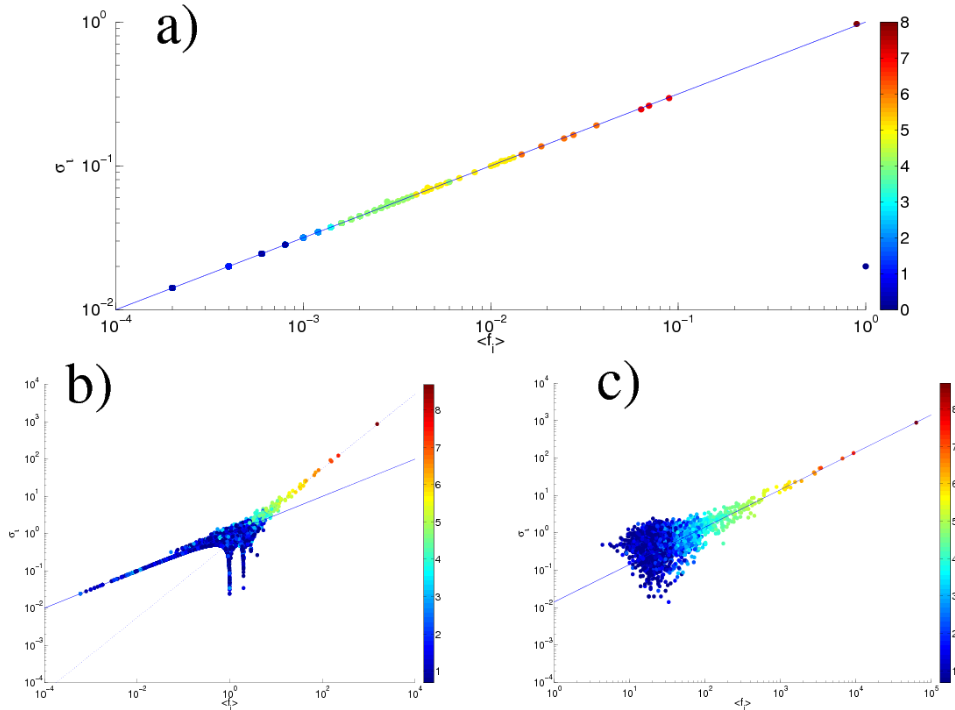

Fig. 2. Flow fluctuation σ as a function of $\langle f \rangle$ from simulations of the numerical traffic model with the traffic aware routing strategy $h = 0.75$ on synthetic scale-free networks with $N = 10^4$ nodes, $K = 37551$ links, and degree distribution $P(k) \sim k^{-2.2}$. Three different time windows are used, respetively: $M = 1$ (panel a), $M = 500$ (panel b) and $M = 10000$ (panel c) Color-coded values represent the logarithm of node degree. The continuous line is the curve $y = x^{0.5}$ while the dashed line is $y \sim x$.

In Fig. 2 we show the results of extensive numerical simulations of the numerical traffic model in the case of the traffic aware routing strategy with $h_d = 0.75$ on a synthetic scale-free network with degree distribution $P(k) \sim k^{-2.2}$. The three panels represent the value of σ_i versus the mean flow on a node $\langle f_i \rangle$ for three different values of the time window M representing respectively the minimal [$M = 1$, panel (a)], an intermediate [$M = 500$ panel (b)] and a high [$M = 10000$ panel (c)] time resolution of the system. Our results highlight the agreement between the theoretical prediction of eq. 7 and the numerical simulations even when they are implemented on a more complex and realistic traffic model. In fact, fig. 2(a) corresponds to the choice of parameters for which $\alpha = 1/2$ for all the nodes in the network independently of their degrees. In contrast, in fig. 2(c) the value of M leads to a linear scaling ($\alpha = 1$) for all the nodes. The situation in fig.2(b) is different, as there is not a single exponent for every node in the network but a crossover from $\sigma \sim \langle f \rangle^{0.5}$ for poorly connected nodes to $\sigma \sim \langle f \rangle$ for the most connected ones. Note that results in fig 2 have been obtained for high traffic values ($p = 7$) as they are necessary for the traffic aware routing protocol to be effective. This latter point demonstrates that the relation of eq.7 is insensitive to the traffic conditions.

Another assumption made in the RD model is that the number of packets introduced in the system each time step is fixed or uniformly distributed around a mean value p. Thus we test the dependency of flow fluctuations on packets creation distribution. Specifically, we focus on both short and heavy tailed packets creation distributions as exponential and power law functions in addiction to the uniform one. To assure the same traffic levels for our simulations all distributions are characterized by the same $\langle p \rangle$ average mean value.

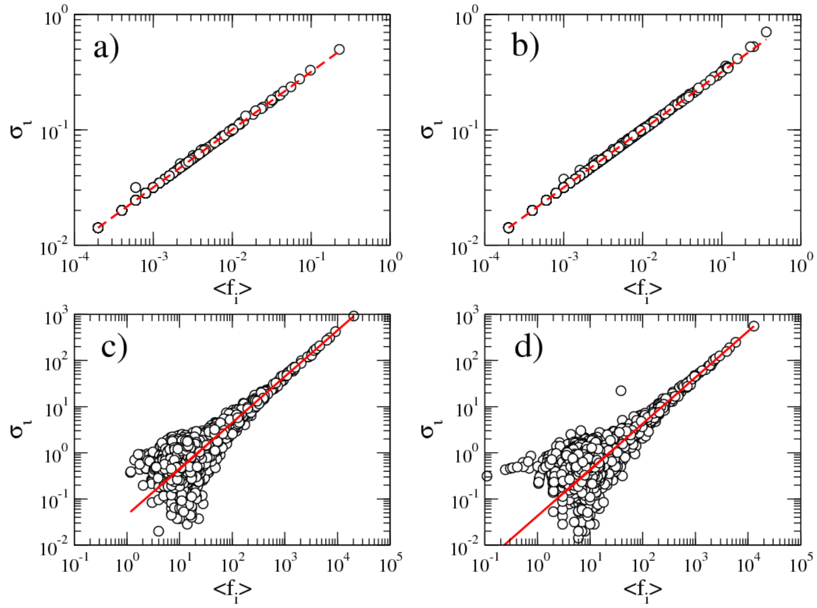

Fig. 3. Flow fluctuation σ as a function of $\langle f \rangle$ from simulations of the numerical traffic model with different packets creation distribution on top of the AS topology networks with $N = 11174$ nodes: a) and c) exponential packets creation distribution with $M = 1$ and $M = 15000$ respectively, b) and d) power law packets creation distribution with $M = 1$ and $M = 15000$ respectively. The dashed line is the curve $y = x^{0.5}$ while the continuous line is $y \sim x$.

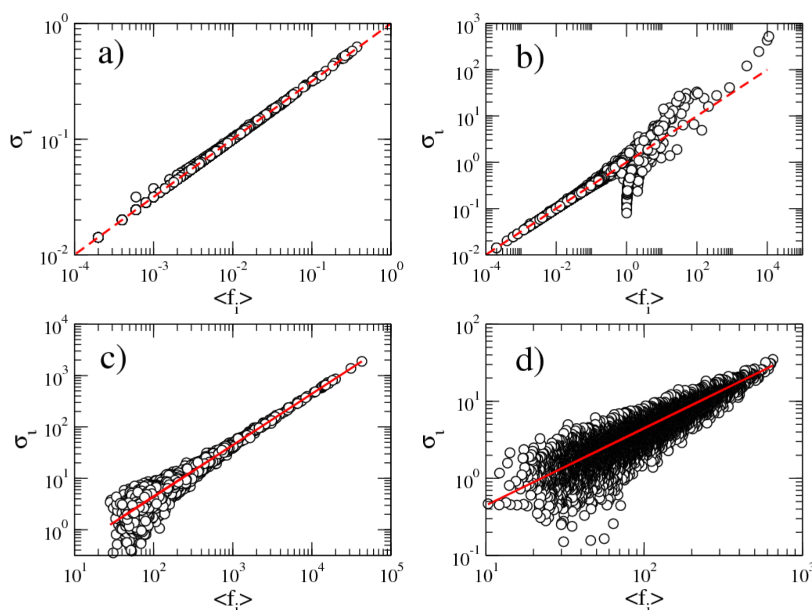

Fig. 4. Flow fluctuation σ as a function of $\langle f \rangle$ from simulations of the numerical traffic model on top of the different topologies with $N = 11174$ nodes: a) and c) Barabási-Albert scale-free network with $M = 1$ and $M = 15000$ respectively, b) and d) Erdös and Rényi random network with $M = 1$ and $M = 15000$ respectively. The dashed line is the curve $y = x^{0.5}$ while the continuous line is $y \sim x$.

Fig. 3 presents the results of the numerical traffic model for two different packets generation distributions and two different values of M. Figs.3(a) and (c) depict the relationship between the mean flux and its variations for an exponential distribution for the packets creation with mean value $p = 2$. Also in this case the two scalings $\alpha = 1/2$ and $\alpha = 1$ hold for the two extreme values of M while the crossover behavior (not shown here) is also observed for intermediates M values. In figs.3(b) and (d) the results for a power

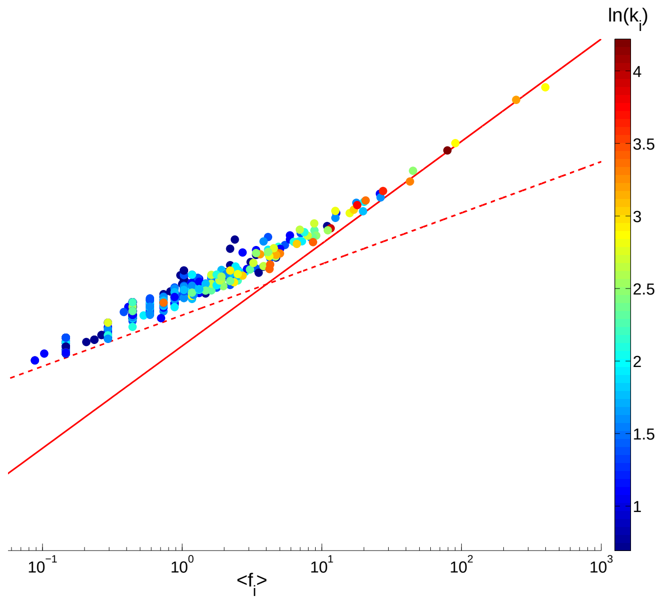

Fig. 5. Flow fluctuation σ as a function of $\langle f \rangle$ from NS-3 simulations for a SF random network with $N = 10^3$ nodes and $K \sim 4 \cdot 10^3$ with constant packets creation rate and the highest data collection resolution made possible by the simulator.

law distribution of packets creation are presented drawing the same conclusions.

In our previous simulations we have used highly heterogeneous networks [synthetic SF networks and the Internet Autonomous Systems (AS) graph] as the substrate topologies but, since eq.7 only depends on the degree of each node and the total number of links, the former results can be extended to any kind of network. Now we show the results for two other types of random networks: Barabási-Albert (BA) SF networks and Erdös-Rényi (ER) graphs (which are characterized by a poissonian degree distribution). Fig.4 reports the value of σ versus $\langle f_i \rangle$ for BA [figs 4(a) and (c)] and ER networks [figs 4(b) and (d)] respectively. Also in this case our theoretical prediction holds, assuring its validity regardless of the underlying topology.

4. Realistic network traffic simulations

To further support the validity of our theoretical prediction in more realistic numerical simulations we conclude our analysis by exploring the effects of degree heterogeneity in real traffic data from communication networks. In our previous work [Meloni *et al.*, 2008] we analyze real traffic data from the Abilene-2 research network[1] and we highlighted the effects of different time windows on the results. Unfortunately, due to the limited size of the network and the low heterogeneity in the degree of the nodes we were unable to show the dependency of the flow fluctuations with nodes degree. The unavailability of sufficient quantity of data from large real networks forces us to employ a computer network simulation used in computer science research: *NS-3*[2]. The Network Simulator (NS) in its version 3.0 is a discrete event simulator targeted at networking research. It allows to fully reproduce all the features of a real computer network, implementing the standard TCP/IP protocols stack and several types of connection media (including traffic control strategies, transmission delays and errors). Thanks to *NS-3* we were able to simulate the behavior of a real large router network with all the protocols used in everyday traffic control. We use as underlying topology a SF random network composed by $N = 10^3$ nodes and $K \sim 4 \cdot 10^3$ links. To see the effects of network topology on flow fluctuations we ran the *NS-3* simulator with randomly chosen origins and destinations at a constant creation rate ($\delta = 0$ no fluctuations in packets creation) and with the entire TCP/IP stacks protocols enabled. While the simulations ran, we collect the flow data on each node with the highest possible

[1] Data publicly available at http://www.internet2.edu/network/
[2] Freely available at http://www.nsnam.org/

resolution (about 10^{-3} seconds) which is comparable with the time scale of traffic dynamics (about 10^{-4} seconds). Then, we computed for each node the mean flow $\langle f_i \rangle$ and its corresponding fluctuations σ_i. The results, presented in Fig. 5, show that all exponents α between $1/2 \leq \alpha \leq 1$ are recovered (as predicted by the theoretical model) and, as no external fluctuations in the packets creation rate are present and the sampling time scale is comparable to that of the dynamics, the observed behavior can be considered as the truly effect of nodes degree heterogeneity.

5. Conclusions

In summary, in this work we analyzed the validity of an analytical law for the prediction of the fluctuations in the mean traffic in a network in realistic conditions and highlighted the effect of degree heterogeneity on the forecasts. We recalled a theoretical model based on random diffusion that depends on three main factors: one related to the dynamics, one related to the topology, and one of purely statistical nature. In this context we checked the validity of theoretical predictions, thanks to extensive numerical simulations, in a series of more realistic and accurate scenarios. We demonstrate the robustness of the predicted law in presence of different routing mechanisms ranging from the simple shortest path routing scheme to an adaptive traffic aware protocol. Differences in the distribution of the packets creation rate have also been analyzed including exponential and heavy-tailed laws confirmed the stability of our predictions. Finally, to emphasize the pure effects of topology on flow fluctuations, that have been elusive untill now, we set up a realistic numerical analysis through an event based network traffic simulator. Our study confirms that in all cases the theoretical prediction, although obtained by means of a very simplified model, holds even in the most realistic environments. Therefore, our results can have high relevance in the design and control of real communication networks.

Acknowledgments

We thank A. Arenas and J. Duch for helpful comments and discussions on this work. S.M. is supported by the Ministerio de Ciencia e Innovación (MICINN) through project FIS2009-13364-C02-01. J.G.G. is supported by the MICINN through the Ramón y Cajal program and grants FIS2008-01240 and MTM2009-13838. Y. M. is supported by the MICINN through projects FIS2008-01240 and FIS2009-13364-C02-01 and by the Government of Aragón (DGA) through a grant to FENOL Group.

References

Albert, R. & Barabási, A.-L. [2002] "Statistical mechanics of complex networks," *Rev. Mod. Phys.* **74**, 47.

Albert, R., Jeong, H. & Barabási, A.-L. [2000] "Error and attack tolerance of complex networks," *Nature* **406**, 378.

Argollo de Menezes, M. & Barabási, A. L. [2004] "Fluctuations in Network Dynamics," *Phys. Rev. Lett.* **92**, 028701; "Separating Internal and External Dynamics of Complex Systems," *Phys. Rev. Lett.* **93**, 068701.

Boccaletti, S., Latora, V., Moreno, Y., Chavez, M. & Hwang, D.U. [2006] "Complex networks: Structure and dynamics," *Phys. Rep.* **424**, 175.

Buldyrev, S.-V., Parshani, R., Paul, G., Stanley, E.-H. & Havlin, S. [2010] "Catastrophic cascade of failures in interdependent networks," *Nature* **464**, 1025.

Caretta Cartozo, C. & De Los Rios, P. [2009] "Extended navigability of small world networks: exact results and new insights.," *Phys. Rev. Lett.* **102**, 238703.

Crucitti, P., Latora, V. & Marchiori, M. [2004] "Error and attack tolerance of complex networks ," *Physica A* **340**, 388.

Crucitti, P., Latora, V. & Marchiori, M. [2004] "A model for cascading failures in complex networks ," *Phys. Rev. E* **69**, 045104(R).

De Martino, D., Dall'Asta, L., Bianconi, G. & Marsili, M. [2009] "Congestion phenomena on complex networks," *Phys. Rev. E* **79**, 015101(R).

Dorogovtsev, S.N., Goltsev, A.V. & Mendes, J. F. F. [2008] "Critical phenomena in complex networks," *Rev. Mod. Phys.* **80**, 1275.

Duch, J. & Arenas, A. [2006] "Scaling of fluctuations in traffic on complex networks," *Phys. Rev. Lett.* **96**, 218702.

Echenique, P., Gómez-Gardeñes, J. & Moreno, Y. [2004] "Improved Routing Strategies for Internet Traffic Delivery," *Phys. Rev. E* **70**, 056105.

Echenique, P., Gómez-Gardeñes, J. & Moreno, Y. [2005] "Dynamics of Jamming Transitions in Complex Networks," *Europhy. Lett.* **71**, 325.

Eisler, Z., Bartos, I., Kertesz, J. [2008] "Fluctuation scaling in complex systems: Taylor's law and beyond," *Adv. in Physics* **57** 89-142.

Gómez-Gardeñes, J. & Latora, V. [2008] "Entropy rate of diffusion processes on Complex Networks," *Phys. Rev. E* **78** 065102(R).

Guimera, R., Díaz-Guilera, A., Vega-Redondo, F., Cabrales, A. & Arenas, A. [2002] "Optimal network topologies for local search with congestion," *Phys. Rev. Lett.* **89**, 248701.

Guimera, R., Arenas, A., Díaz-Guilera, A. & Giralt, F. [2002] "Dynamical properties of model communication networks," *Phys. Rev. E* **66**, 026704.

Jacobson, V. & Karels, M. [1988] "Congestion Avoidance and Control," *ACM SIGCOMM Computer Communication Review* **25** (1).

Kurose, J. & Ross, K. [2008]. *Computer Networking - A Top-Down Approach (4thth ed.)*. (Addison Wesley).

Meloni, S., Gómez-Gardeñes, J., Latora, V. & Moreno, Y. [2008] "Scaling breakdown in flow fluctuations on complex networks," *Phys. Rev. Lett.* **100**, 208701.

Meloni, S. & Gómez-Gardeñes, J. [2010] "Local Empathy provides Global minimization of Congestion in communication networks," *Phys. Rev. E* **82**, 056105.

Motter, A.-E. & Lai, Y.-C. [2002] "Cascade-based attacks on complex networks," *Phys. Rev. E* **66**, 065102(R).

Newman, M. E. J. [2003] "The Structure and Function of Complex Networks," *SIAM Rev.* **45**, 167.

Noh, J.D. & Rieger, H. [2004] "Random Walks on Complex Networks," *Phys. Rev. Lett* **92**, 118701.

Ohira, T. & Sawatari, R. [1998] "Phase transition in a computer network traffic model," *Phys. Rev. E* **58**, 193.

Pastor-Satorras, R., Vázquez, A. & Vespignani, A. [2001] "Dynamical and correlation properties of the Internet.," *Phys. Rev. Lett.* **87**, 258701.

Pastor-Satorras, R. & Vespignani, A. [2004] *Evolution and Structure of the Internet*, (Cambridge University Press, Cambridge, UK).

Rosato, V., Issacharoff, L., Meloni, S., Caligiore, D. & Tiriticco, F. [2008] " Is the topology of the Internet network really fit to sustain its function?," *Physica A* **387**, 1689.

Serrano, M. A., Boguñá, M., & Díaz-Guilera, A. [2005] "Competition and Adaptation in an Internet Evolution Model," *Phys. Rev.Lett.* **94**, 038701.

Solé, R. V. & Valverde, S. [2001] "Information transfer and phase transitions in a model of internet traffic," *Physica A* **289**, 595.

Sreenivasan, S., Cohen, R., Lopez, E., *et al.* [2007] "Structural bottlenecks for communication in networks,"*Phys. Rev. E* **75**, 036105.

Tadic, B., Rodgers, G.J. & Thurner, S. [2007] "Transport on Complex Networks: Flow, Jamming and Optimization," *Int. J. Bifurcation and Chaos (IJBC)* **17**, 7, 2363-2385.

Wang, B.H. & Zhou, T. [2007] "Traffic Flow and Efficient Routing on Scale-Free Networks: A Survey," *J. Korean Phys.Soc.* **50**,134.

Yan, G., Zhou, T., Hu, B., Fu, Z.-Q. & Wang, B.-H. [2006] "Efficient routing on complex networks," *Phys. Rev. E* **73**, 046108.