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Robustness of cultural communities in an open-ended Axelrod's model

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HIGHLIGHTS

- An Axelrod-like model describes the evolution of topics in the social debate.
- The introduction of new topics has little effect on cultural groups.
- Renewal of topics influences substantially cultural overlaps.

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ABSTRACT

We consider an open-ended set of cultural features in the Axelrod model of cultural dissemination. By replacing the features in which a high degree of consensus is achieved by new ones, we address here an essential ingredient of societies: the evolution of topics as a result of social dynamics and debate. Our results show that, once cultural clusters have been formed, the introduction of new topics into the social debate has little effect on them, but it does have a significant influence on the cultural overlap. Along with the Monte Carlo simulations, we derive and numerically solve an equation for the stationary cultural overlap based on a mean-field approach which reproduces the qualitative behavior of the model.

1. Introduction

Agent-Based Modeling has become one of the major techniques to study complex adaptive systems, being currently a paradigm in fields as diverse as ecology, sociology, economics or engineering. The use of agent-based models (ABM) [1,2] in the study of social phenomena provides a powerful theoretical framework that gives useful insights about the fundamental mechanisms at work in social systems. In ABMs, agents represent interacting entities (for example, individuals or groups of individuals) and are characterized by a set of internal states. In particular, in opinion ABMs, agents are provided with a set of opinion variables [3]. In 1977, Axelrod [4] proposed an ABM for the dissemination of culture based on the idea of homophily, *i.e.*, the tendency of individuals to interact with similar ones and, as a consequence, become even more alike. According to this idea, the likelihood for an individual to imitate a cultural trait from another individual will depend on how many other traits they have already in common. For low values of the initial cultural diversity, the resulting dynamics converges

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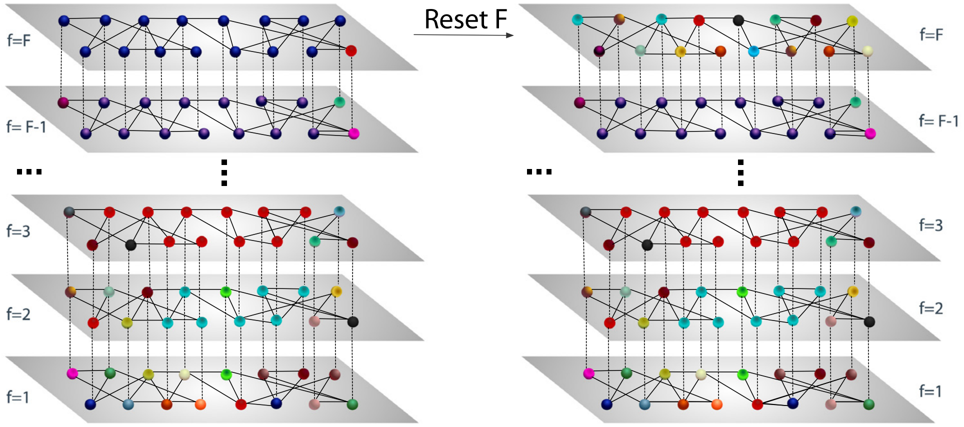

Fig. 1. In this illustrative figure, each layer represents a different feature ($f = 1, 2, \ldots, F$), while nodes represent the agents. Each agent is depicted by the same node in all the layers, and links stand for the contacts between agents. When the fraction of agents sharing the most abundant trait of a feature reaches the value φ (left panel, layer F), consensus on the topic is assumed and it is replaced by a new emerging topic through the initialization of traits in layer F (right panel). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

to a global monocultural state, characterized by agents that share every cultural trait. In contrast, for high values of initial diversity multiculturality prevails. This change of macroscopic behavior has been characterized as a non-equilibrium phase transition [5–7]. In fact, the use of methods and tools of statistical physics for describing the spread of cultural traits, opinions, or conventions in terms of non-equilibrium phase transition is nowadays well established [3]. In particular, this approach has proved to be effective for describing ordering dynamics that generate global consensus as an emergent phenomenon in systems characterized by a big number of interacting agents, and nowadays the number of different models that present such ordering transition is extensive. For example, we can cite the opinion dynamics of the simple Sznajd model [8], or systems where individuals adopt the local majority state [9], or approaches based on a nonlinear interaction between opinion vectors [10]. Similar models have also been used for the description of linguistic dynamics, as for the Naming game models [11,12], or for application to market phenomena, as in the case of the Minority Game [13].

Turning back to the Axelrod's Model, we can cite several studies which focus on specific issues of this model, such as the effects of the network's topology [14,15], clustering [16], cultural drift (modeled as noise) [17,18], local social pressure [19], confidence thresholds [20], media (represented by an external field) [21,22], mobility and segregation [23–25], and dynamic networks [26]. In addition to the Axelrod model, other types of dynamics for vectors of opinions have been proposed, including binary [27–29] and continuous variables [30] for the opinions, as well as multilayer structures [31].

Although the Axelrod model can capture some realistic features of societies [32], it does not take into account a key characteristic of real-world social dynamics, namely, the evolution of topics in the social debate. This fact defines an open-ended system where new themes enter the social debate while older ones are archived. While, for example, in the nineteenth century slavery was discussed and in the first half of the twentieth century there was a debate on women's suffrage, currently these themes are not any more at debate. Instead, new issues arise and become the center of the political discourse.

In this work, we consider a model that takes into account the open-ended nature of the social debate. This particular aspect of social dynamics has been previously dealt with in other ABM used to describe the exchange of linguistic conventions [33–35]. In the case of the Axelrod model, an open set of cultural features is easily introduced by substituting the topics which achieve a high degree of consensus. This is implemented reinitializing with random traits the cultural feature that achieves a level of agreement greater than a threshold φ. The parameter φ can be interpreted as the resistance of the society, that is, the minimum level of agreement required to assume consensus on an issue. Our numerical results show that the emergence of new topics for discussion into the social debate has little effect on cultural groups once they have been consolidated, but it does have a considerable effect on cultural overlaps. Along with Monte Carlo simulations, we have also performed a mean-field analysis. Although the mean-field approach reproduces qualitatively some aspects of the numerical results, it substantially underestimates the value of the cultural overlap, a fact that highlights the influence of the topology and the correlations between the different cultural features in the Axelrod dynamics.

2. Renewal of social debate topics in the Axelrod model

In Axelrod's original model of cultural dissemination, N cultural agents occupy the nodes of a network whose links define the social contacts among them. Each agent i is assigned to a culture modeled as a vector of F integer variables $\{\sigma_f(i)\}$ ($f = 1, \ldots, F$), the *cultural features*, that can assume q values, $\sigma_f = 0, 1, \ldots, q-1$, the *traits* of the feature. The features of each agent i are initialized by random assignment of traits from a uniform distribution. The parameter q represents the initial cultural diversity. At each time step, a random agent i is chosen and allowed to imitate an unshared feature trait of a

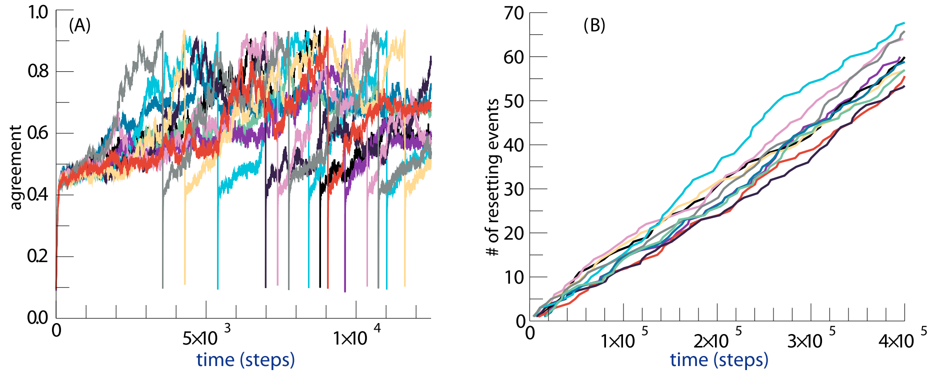

Fig. 2. (A) Time evolution of the agreement level, *i.e.*, fraction of agents sharing the most abundant trait, on a given feature. When the level of agreement reaches the value $\varphi = 0.95$, the feature is reinitialized by random assignment of traits. (B) Cumulative number of resetting events (features reinitializations) as a function of time, for different features. The panels show the evolution of a representative realization in a random regular network of degree $k = 6$. Different colors represent different features. Other values are $q = F = 10$, $N = 10^3$. See the text for further details. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

randomly chosen neighbor j, with a probability proportional to their cultural overlap ω_{ij}, which is defined as the fraction of common cultural features,

$$\omega_{ij} = \frac{1}{F} \sum_{f=1}^{F} \delta_{\sigma_f(i),\sigma_f(j)} \,, \tag{1}$$

where $\delta_{x,y}$ is the Kronecker's delta defined as $\delta_{xy} = 1$ if $x = y$ and $\delta_{xy} = 0$ otherwise.

In this work, we consider the incorporation of new topics into the social debate as consensus is reached in other topics, modeling this situation through the reinitialization of the features in which a given level of agreement has been reached. Explicitly, when the proportion of agents sharing the most abundant trait of a feature f exceeds a threshold φ $(0 < \varphi \leq 1)$, consensus on the topic is assumed and the topic is replaced by a new one. To this end, see Fig. 1, the feature f of each agent is drawn randomly from a uniform distribution on the integers $\{0, 1, \ldots, q-1\}$. The parameter φ (here called resistance) represents the minimum level of agreement required to assume consensus on a topic. Note that for $\varphi = 1$ Axelrod's original model is recovered.

3. Results and discussion

In this section we present the numerical results of our Monte Carlo simulations along with analytical results obtained for a mean-field approximation. In order to compare the numerical results with the analytical ones, for the simulations we consider the case of random regular networks. Random regular networks are random networks of fixed degree k, which means that all nodes are equivalent [36]. However, for completeness, at the end of this section we present some results for regular lattices, which allow us to compare the behavior of the proposed model with that of Axelrod's original model. The results shown below are obtained by averaging over a large number (typically 100) of networks and different initial conditions.

The system is initialized by a random assignment of agent's cultures, that is, for every node in the network the F features are drawn randomly from a uniform distribution on the set of integers $\{0, 1, \ldots, q-1\}$. The process is stopped when the system reaches a stationary state, characterized by quasi-constant values of the observables between resetting events. Note that, while Axelrod's original model is characterized by two types of frozen states in which all the pairs of agents have overlap either 1 or 0, the dynamics here proposed prevents the monocultural frozen state. Nevertheless, the simulations of the proposed dynamics show a cyclical behavior after a limited number n of resetting events, n being of the order of magnitude of F. After n resetting events all the observables take, within a range of fluctuations, the same value at the time prior to each resetting event. In our case, we run the dynamics a number of steps involving $100 \times n$ resetting events before measuring the observables.

To illustrate the dynamics proposed here, Fig. 2 displays the time evolution of a characteristic realization for $F = 10$, $q = 10$, and a value of $\varphi = 0.95$. Panel (A) shows the evolution of the level of agreement on each feature, *i.e.*, the fraction of agents sharing the most abundant trait of a given feature. Different colors represent the different features $f = 1, 2, \ldots, F$. As shown, when the level of agreement on a feature f reaches the value $\varphi = 0.95$, the feature f is initialized by assigning at random a new value to the corresponding cultural component in the cultural vectors of all the agents. This leads to a value close to $1/q$ for the level of agreement on the reset feature. Panel (B) shows the time evolution of the cumulative number of features initializations; each line represents a different feature. As it can be seen, the symmetry of the dynamics, according

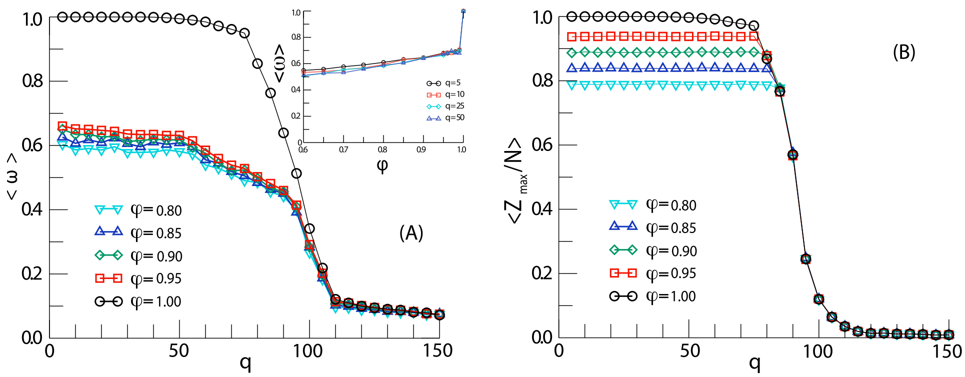

Fig. 3. (A) Average cultural overlap $\langle\omega\rangle$ in the steady state versus the number of traits q, for different values of the resistance φ. The inset shows the values of the overlap versus the resistance φ, for different values of the number of traits q. (B) Fraction of agents sharing the most abundant trait $\langle Z_{max}/N\rangle$ in the steady state versus the number of traits q, for different values of the resistance φ. Other values are $F = 10$, $N = 10^3$. Each point is obtained by averaging over 100 independent realizations in random regular networks of degree $k = 6$. All standard errors are below 5%.

to which there are not prevalent features, entails a similar evolution of the cumulative number of resetting events for the different features.

The usual order parameter for Axelrod's original model is S_{max}/N, where S_{max} is the average number of agents of the most abundant culture. Large values (close to unity) of the order parameter represent cultural globalization. In particular, in the ordered state ($S_{max}/N = 1$) all the agents belong to the same cultural group, that is, they share all the cultural traits. Nevertheless, the model here proposed ($\varphi < 1$) precludes this monocultural state. Actually, due to the open nature of the cultural features it is no longer possible to characterize cultures through unanimous consensus on all the topics, being more convenient to impose a less restrictive condition. In this sense, the cultural overlap averaged over all the links $\langle\omega\rangle$ constitutes a measure of multiculturalism. The averaged overlap $\langle\omega\rangle$ is defined as:

$$\langle\omega\rangle = \frac{1}{E}\sum_{i}\sum_{j\neq i} A_{ij}\omega_{ij}\ , \tag{2}$$

where E is the number of links, A_{ij} is the element (i, j) of the adjacency matrix ($A_{ij} = 1$ if i and j are linked and 0 otherwise), and ω_{ij} is the cultural overlap of agents i and j defined in (1). Large values of the average overlap ($\langle\omega\rangle \sim 1$) correspond to a state close to monoculturalism, while low values ($\langle\omega\rangle \sim 0$) correspond to multiculturalism. In Panel (A) of Fig. 3, we plot the average cultural overlap $\langle\omega\rangle$ as a function of the initial cultural diversity q, for different values of the resistance φ. The behavior of the parameter $\langle\omega\rangle$ strongly depends on q. For large q, corresponding to the disordered phase, it displays low values which are not dependent on φ. In contrast, for small q, the ordered phase is strongly influenced by the resetting mechanism. In fact, we can observe that simulations with $\varphi < 1$ present a smaller overlap in relation to the one corresponding to Axelrod's original model ($\varphi = 1$). This fact implies that the emergence of new themes in the social debate generate a strong impact on cultural overlap, causing a decrease in its values. We can better characterize this behavior plotting the average overlap as a function of φ for fixed q values corresponding to the ordered phase. As can be appreciated in the inset of Fig. 3, the overlap presents a weak linear growth in dependence of φ, with an abrupt jump in correspondence of $\varphi = 1$. This intriguing behavior will be explored in more detail at the end of the section by means of a mean-field approximation.

As a complementary observable, we have also computed the fraction of agents sharing the most abundant cultural trait, Z_{max}/N, which measures partial opinion convergence. Here Z_{max} stands for the number of agents that share the most common trait σ_f of the feature f with the highest level of agreement:

$$Z_{max} = max\{max\{\sum_{j\neq i}\delta_{\sigma_f(i),\sigma_f(j)} : i = 0, 1, \ldots, N\} : f = 0, \ldots, F-1\} \tag{3}$$

In Panel (B) of Fig. 3, we plot Z_{max}/N as a function of the initial cultural diversity q, for different values of the resistance φ. As it is shown, the only effect of the features resetting in the partial opinion convergence is limited to low values of q (those corresponding to the ordered state in Axelrod's original model). On the other hand, this discrepancy for low values of q is almost the minimum difference that the constraint imposed by the value of φ allows. This means that the incorporation of new topics into the social debate has a minimal effect on the convergence of the rest of the topics in which there is already a high degree of consensus. In fact, surprisingly, the stationary value of Z_{max}/N, at low q values, is very close to the set φ value. This fact suggests that, in the ordered phase, it always exists at least one cultural trait so pervasive to present a partial convergence very close to the highest value allowed by the resetting mechanism.

The small effect of the emergence of new debate topics on the rest of the cultural features of the agents makes it of further interest to study how it affects the dynamics of cultural groups. In Axelrod's original model, cultural domains are composed of agents that share all the traits, that is, two agents i and j belong to the same cultural domain if, and only if, their corresponding

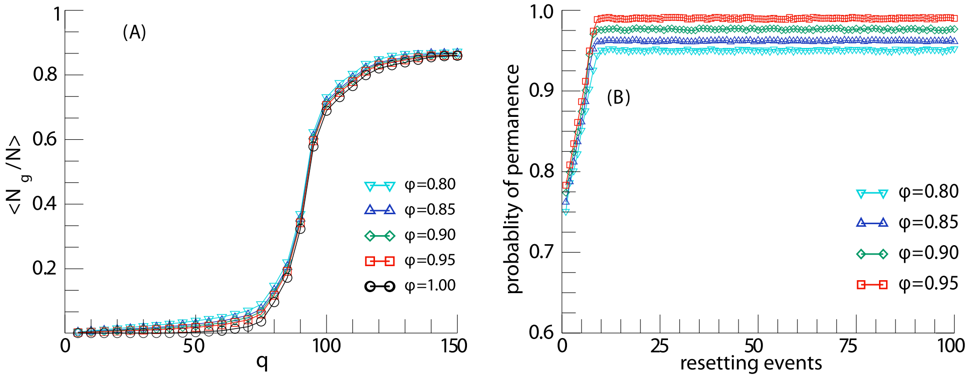

Fig. 4. (A) Scaled number of different cultural groups $\langle N_g/N\rangle$ versus the number of traits q, for different values of the resistance φ. (B) Evolution of the probability for an agent to remain in the same cultural group between two consecutive reset events, for different values of the resistance φ and for a number of traits $q = 80$. Here, two agents belong to the same cultural group if they share, at least, $F-1$ cultural traits. Other values are $F = 10$, $N = 10^3$. Each point is obtained by averaging over 100 independent realizations in random regular networks $k = 6$. All standard errors are below 2%. See the text for further details.

cultural vectors are equal $\{\sigma_f(i)\} = \{\sigma_f(j)\}$ for all $f \in \{1, 2, \dots, F\}$. As exposed above, the model here proposed ($\varphi < 1$) bans the formation of homogeneous cultural domains and it is more convenient to impose a less restrictive condition. To this end, we relax the definition of cultural groups by allowing that two agents belong to the same cultural group if they share, at least, $F - 1$ cultural traits. According to the new condition, the resetting of a feature should not lead to the disintegration of all the groups so defined. Note that previous condition is not transitive, that is, if agents i and j disagree on a trait u, while agents j and k disagree on a different trait v, agents i and k disagree on traits u and v and cannot belong to a common cultural group. Consequently, to establish a partition, we apply the following algorithm: first, we consider the largest set in which all the agents share the same $F - 1$ cultural traits. Once a cultural group is established according to the previous criterion, we consider the rest of the agents and repeat the same process until all the agents are assigned to a cultural group (which can be unitary).

The observable $\langle N_g/N\rangle$, where N_g is the number of cultural groups (as defined above) in the final state, provides a measure of the disorder of the system [21].

Fig. 4 shows the numerical results for the cultural groups, as defined above, on a random regular network of size $N = 1000$ and degree $k = 6$. In panel (A) of Fig. 4 we show the average fraction of different cultural groups $\langle N_g/N\rangle$ as a function of the initial cultural diversity q, for different values of the resistance φ. As it can be seen, the resistance has little influence on the number of cultural groups in the steady state, as it is expected that the resetting process does not have a strong effect on the dynamics of the cultural groups. We also show in Panel (B) of Fig. 4 the time evolution of the probability p that an agent remains in the same cultural group between two consecutive reset events, for a number of traits ($q = 80$) corresponding to the transition between ordered and disordered phases and for different values of the resistance φ. As it is shown, there is a transient where the probability to remain in the same group increases with time. This transient corresponds to a number of resets equal to the number of cultural features F. After this transient, the permanence probability p is constant and close to one ($p > 0.95$ for resistance values $\varphi \geq 0.8$). This means that, once the cultural groups have been consolidated, the emergence of new topics in the social debate does not have a strong effect on them. Furthermore, the higher the resistance, the greater the probability of permanence and, therefore, the less influence of the renewal of topics on the structure of the cultural groups.

In order to explore the abrupt transition between full and partial consensus phases at $\varphi = 1$, we take advantage of the mean-field approximation, implemented by Castellano et al. in [5]. Strictly following their approach, we can compute the average overlap:

$$\langle \omega(t)\rangle = \sum_{m=1}^{F} \frac{m}{F} P_m(t)\ . \tag{4}$$

Here $\langle\,\rangle$ stands for the ensemble average and $P_m(t)$ denotes the probability that a random link connects two agents that agree on m topics, that is, with overlap m/F.

According to Ref. [5] the time evolution of $P_m(t)$, in the mean-field approximation, is given by the master equation:

$$\frac{dP_m}{dt} = \sum_{u=1}^{F-1} \frac{u}{F} P_u \Bigg\{ \delta_{m,u+1} - \delta_{m,u} + (k-1) \sum_{n=0}^{F} (P_n W_{n,m} - P_m W_{m,n}) \Bigg\}\ , \tag{5}$$

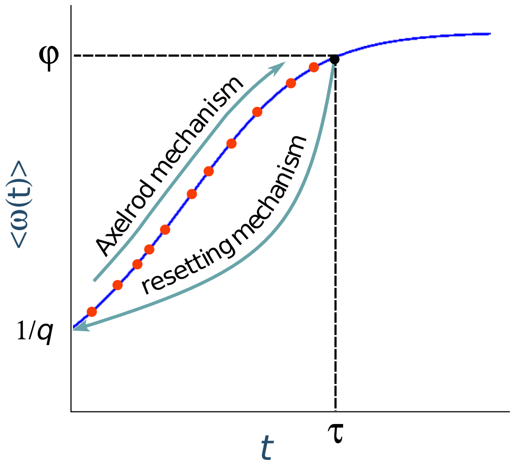

Fig. 5. Schematic representation of the approximation used to compute the average overlap $\langle\omega(t)\rangle$. The blue line represents the mean-field evolution of the average overlap for the Axelrod model. The red dots represents a snapshot of the average overlap for single features which, for the steady state, are randomly distributed along the interval $[0, \tau]$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

where $W_{n,m}$ is the transition probability of a n-type link to a m-type link due to the updating of a neighboring link. As before, δ represents the Kronecker's delta. Neglecting correlations among adjacent links, the second-order transition probabilities are given by:

$$W_{n,m} = \frac{F-n}{F} \langle\omega(t)\rangle \, \delta_{m,n+1} + \frac{n}{F}\delta_{m,n-1} \ . \tag{6}$$

Using Eqs. (4)–(6) we can obtain P_m integrating the differential equation:

$$\begin{aligned} \frac{dP_m}{dt} &= \sum_{u=1}^{F-1} \frac{u}{F} P_u \Bigg\{ \delta_{m,u+1} - \delta_{m,u} \\ &+ (k-1)\sum_{n=0}^{F} P_n \left(\frac{F-n}{F} \sum_{i=1}^{F} \frac{i}{F} P_i \, \delta_{m,n+1} + \frac{n}{F}\delta_{m,n-1} \right) \\ &- (k-1)\sum_{n=0}^{F} P_m \left(\frac{F-m}{F} \sum_{i=1}^{F} \frac{i}{F} P_i \, \delta_{n,m+1} + \frac{m}{F}\delta_{n,m-1} \right) \Bigg\} \ , \end{aligned} \tag{7}$$

with the initial condition: $P_m(0) = \frac{F!}{m!(F-m)!} \left(\frac{1}{q}\right)^m \left(\frac{q-1}{q}\right)^{F-m}$, which correspond to randomly, unbiased and uncorrelated initial traits. Once obtained the values of P_m, we can compute the average overlap $\langle\omega(t)\rangle$.

Until this point, we did not consider the effect of the resetting mechanism. In order to do that, we first notice that the resetting mechanism induces the mean overlap of a single feature, $\langle\omega_i(t)\rangle$, to behave cyclically. The overlap, which starts at $1/q$, reaches φ as a consequence of the Axelrod mechanism. After that, the resetting mechanism will bring it back to $1/q$ (see Fig. 5) . In the steady state it is reasonable to assume that the features overlaps are randomly distributed between one reset and another. Following these assumptions, the mean overlap considering the resetting mechanism, $\langle\omega_R\rangle$, can be estimated as:

$$\langle\omega_R\rangle \sim \frac{1}{F}\sum_{i=1}^{F}\langle\omega_i(t)\rangle \ \sim \frac{1}{\tau}\int_0^{\tau}\langle\omega(t)\rangle dt \ , \tag{8}$$

where τ corresponds to the t value where $\langle\omega(t)\rangle = \varphi$. In this estimation we considered that, after a long enough transient, the resetting of the different features distributed their overlaps values homogeneously over the time. It follows that the time-averaged value of the overlap of a feature constitutes an estimator of the overlap averaged over all the features.

Fig. 6 displays the average overlap in the stationary state $\langle\omega_R\rangle$ as a function of the resistance φ, for different values of the number of cultural traits q according to the mean-field estimation. As shown, the mean-field estimation reproduces the qualitative behavior of the numerical results, displaying an abrupt transition at $\varphi = 1$. Furthermore, the comparison between numerical results (inset in panel (A) of Fig. 3) and theoretical predictions (Fig. 6) highlights that the mean-field approach underestimates the cultural overlap value. The reasons for this underestimation in the mean-field approximation rely on the assumptions on which it was based. On one hand, in the Axelrod dynamics, cultural clusters are associated to

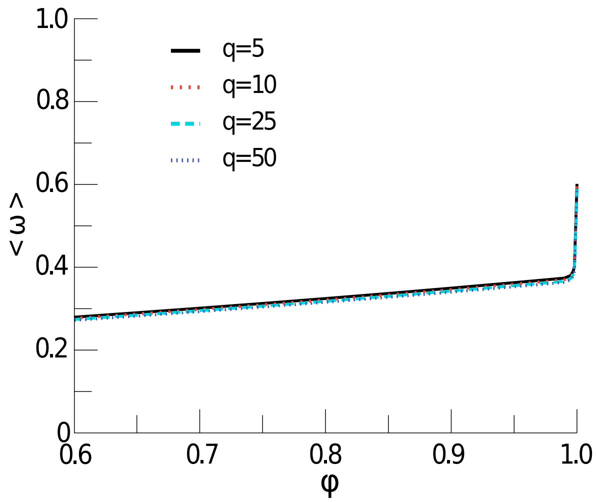

Fig. 6. Mean-field estimation of the average overlap $\langle\omega\rangle$ as a function of the resistance φ, for different values of the number of traits q. As shown, mean-field prediction qualitatively captures the abrupt transition at $\varphi = 1$. Other values are $F = 10$ $k = 6$. See the text for further details.

topological clusters, which are totally absent in the mean-field approximation. On the other hand, the homophily mechanism enhances correlations among the different features, while the mean-field approximation neglects those correlations. Note that these two characteristics of the Axelrod dynamics, namely the formation of cultural clusters associated with the contact network and the correlation between the different cultural features, establish a connection between the formation of cultures and interpersonal relationships. Notwithstanding the quantitative disagreement between MC simulations and the analytical approximation, the latter does capture the behavior of the model, and thus, provides mechanistic hints about what is going on in the system's dynamics.

In order to test the robustness of the proposed dynamics under the election of the network, we have studied the system for a regular lattice ($k = 4$) as in Axelrod's original model. Panels (A) and (B) of Fig. 7 show the same results of Fig. 3, in this case for a lattice. In Panel (A) of Fig. 7, we plot the average cultural overlap $\langle\omega\rangle$ in the stationary state as a function of the initial cultural diversity q, for different values of the resistance φ. In the same way, Panel (B) of Fig. 7 shows the fraction of agents sharing the most abundant cultural trait Z_{max}/N as a function of q, for different values of φ. As can be seen by comparing Figs. 7 and 3, the results are qualitatively similar for both network topologies. However, random networks exhibit a larger critical value of q than latices, highlighting the fact that network heterogeneity promotes cultural convergence, as has been previously observed for the classical Axelrod's model [14,15]. As an additional test of robustness, we have also simulated the system for different number F of cultural features in random regular networks. Panel (C) of Fig. 7 shows the stationary cultural overlap $\langle\omega\rangle$ for $F = 2$, whereas panel (D) shows the same observable for $F = 50$. As shown, regardless of the different nature of the transition [5], the main difference between Axelrod's original model ($\varphi = 1$) and the here proposed modification ($\varphi < 1$) is maintained, that is, the renewal of the cultural features significantly decreases the cultural overlap.

4. Summary and concluding remarks

In the Axelrod model for cultural dissemination, we have considered the incorporation of new topics into the social debate by resetting the features in which the fraction of agents sharing the most abundant trait exceeds a threshold φ. This parameter φ, that we call resistance, represents the minimum level of agreement required to assume consensus on a topic. The introduction of an open-ended set of topics through this resetting mechanism avoids the frozen monocultural state of Axelrod's original model. We have performed extensive numerical simulations which show that, for high enough values of the initial cultural diversity, the dynamics leads to a multicultural society fragmented into clusters characterized by a high degree of cultural agreement. Remarkably, we show that the renewal of the social discussion topics does not have a considerable effect on the distribution of consolidated cultural clusters. This preservation of group cohesion can be consistent with the idea that individuals take a position on emerging issues of social debate in accordance with the trend of the cultural group they belong to. However, this renewal of discussion topics has a significant influence on the cultural overlaps, with the result that cultural clusters are less homogeneous than in the case of a closed set of discussion topics.

In addition, we have performed a mean-field analysis based on two assumptions, namely, the presumption that the agents interact with each other in proportion to their average abundance and the disregard of correlations between the different features. Although the mean-field analysis qualitatively reproduces the numerical results, it yields an underestimation of the mean cultural overlap. This underestimation of the mean-field approximation highlights the key role of the local interactions in the Axelrod dynamics, where cultural and topological clusters are closely linked, as well as the imitation-driven correlations among the different cultural features. Altogether, our work opens the path to consider more sophisticated

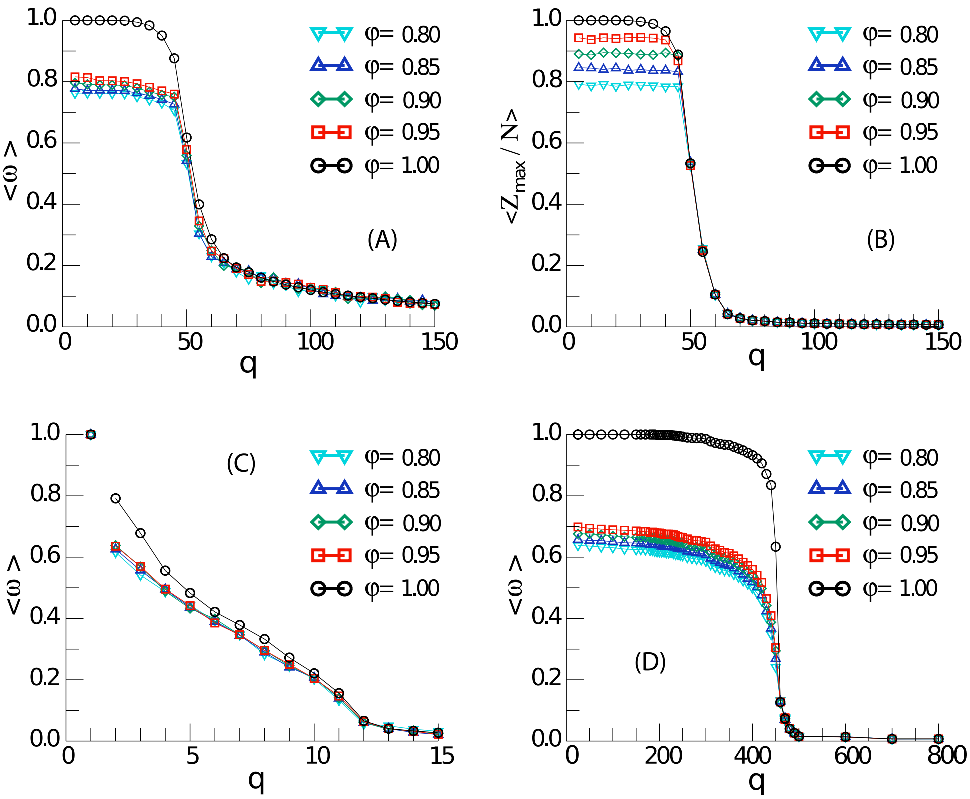

Fig. 7. Panels (A) and (B) show the case when the agents are located in the nodes of a regular lattice ($k = 4$), whereas panels (C) and (D) explore different number F of culture features in a random regular network ($k = 6$). (A) Asymptotic average cultural overlap $\langle\omega\rangle$ versus the number of traits q, for different values of the resistance φ and $F = 10$. (B) Fraction of agents sharing the most abundant trait $\langle Z_{max}/N\rangle$ in the stationary state versus q, for different values of φ and $F = 10$. (C) Average cultural overlap $\langle\omega\rangle$ versus q, for different values of the resistance φ and $F = 2$. (D) Cultural overlap $\langle\omega\rangle$ versus q, for different values of φ and $F = 50$. $N = 10^3$ in all the panels, and each point is obtained by averaging over 100 independent realizations. All standard errors are below 5%. See the text for further details.

models, in which agreement is not frozen once reached and to include higher-order correlations, for instance, by introducing updating rules that involve more than one (possibly correlated) features.

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